A LEVEL MATHEMATICS AT ALL SAINTS CATHOLIC HIGH SCHOOL INTRODUCTION TO A-LEVEL MATHEMATICS

This booklet is to be used as preparation for your A-level Mathematics course. You should have met all topics here at GCSE and you need to make sure you have a good knowledge and understanding of these topics before you start your course in September. It is important that you spend some time working through this booklet to give you a good start to the course. You may not need to do every question on every section.

During the A Level Mathematics course, you will need to have your own more advanced scientific calculator. The model that we strongly recommend is "Casio fx-991EX Classwiz". This can be purchased from all good retailers, priced approximately £22. Alternatively, this can be ordered through All Saints ParentPay priced at £20. If you have already purchased this for use at GCSE, then there is no need to buy a new one. If you have used a lower specification scientific calculator, you will need to upgrade to do the A Level course.

You will need this during your first full week in September. Please ensure you come organised to your first lesson, otherwise you might risk falling behind!

Further help is available from:

- New Head Start to A Level Maths (CGP Workbooks)
- www.corbettmaths.com
- www.mathcentre.ac.uk
- Email m.baker@allsaints.sheffield.sch.uk

ALGEBRA

Collecting like terms



In the example the expression has been rewritten with each set of like terms grouped together, before simplifying by adding/subtracting the like terms. You may well not need to write down this intermediate stage.

Now you try these:-

1. Simplify the following expressions:

(i) 2x + 3y - x + 5y + 4x(ii) 5a - 2b + 3c - 2a + 5b

Multiplying out brackets

Example

Simplify the expressions (i) 3(p-2q) + 2(3p+q)(ii) 2x(x+3y) - y(2x-5y)

Solution

Each term in the bracket must be multiplied by the number or expression outside the bracket.

(i)
$$3(p-2q) + 2(3p + q)$$

= $3p - 6q + 6p + 2q$
= $9p - 4q$
(ii) $2x(x + 3y) - y(2x - 5y)$
= $2x^2 + 6xy - 2xy + 5y^2$
= $2x^2 + 4xy + 5y^2$

Multiplying out two brackets of the form (ax + b)(cx + d) gives a quadratic function. Each term in the first bracket must be multiplied by each term in the second bracket.



Now you try these:-

1. Multiply out the brackets and simplify where possible:

(i)
$$3(2x + 3y)$$
 (ii) $4(3a - 2b) - 3(a + 2b)$ (iii) $p(2p - q) + 2q(p - 3q)$

- 2. Multiply out these expressions.
 - (i) (x + 1)(x 3) (ii) (x + 2)(2x + 1) (iii) (x 3)(x 4) (iv) (3x + 2)(x 2)

Factorising

Algebraic expressions

To factorise an expression, look for numbers and/or letters which are common factors of each term. We often talk about "taking out a factor" – this can cause confusion as it tends to make you think that subtraction is involved. In fact you are, of course, dividing each term by the common factor, which you are "taking out".

Example

Factorise the following expressions.

- (i) 6a + 12b + 3c
- (ii) $6x^2y 10xy^2 + 2xy$

Solution

- (i) 3 is a factor of each term. 6a + 12b + 3c= 3(2a + 4b + c)
- (ii) 2xy is a factor of each term. $6x^2y - 10xy^2 + 2xy$ = 2xy(3x - 5y + 1)

Check your answers by multiplying out the brackets.

Now you try these:-

- 1. Factorise the following expressions:
 - (i) 10ab + 5ac
 - (ii) $2x^2 + 4xy 8xz$
 - (iii) $3s^2t 9s^3t + 12s^2t^2$

Factorising quadratics

To factorise a simple quadratic of the form $x^2 + bx + c$ The method is:

- 1. Form two brackets (x....)(x....)
- 2. Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example

Factorise $x^2 - 9x - 10$

Solution

We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1. So $x^2 - 9x - 10 = (x - 10)(x + 1)$ To factorise a quadratic of the form $ax^2 + bx + c$ The method is:

- 1. Find two numbers that multiply together to make ac and add to make b
- 2. Split up the bx term using the numbers found in step 1.
- 3. Factorise the front and back pair of expressions as fully as possible.
- 4. There should be a common bracket. Take this out as a common factor.

Example

Factorise $6x^2 + x - 12$

Solution

We need to find two numbers that multiply to make $(6 \times -12 =) -72$ and add to make 1. These two numbers are -8 and 9.

Therefore,
$$6x^2 + x - 12 = 6x^2 - 8x + 9x - 12$$

= $2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)
= $(2x + 3)(3x - 4)$

Factorising quadratics of the form $x^2 - a^2$ (difference of two squares)

<u>Remember</u> $x^2 - a^2 = (x+a)(x-a)$

Examples		
Factorise	(i) (ii)	$x^2 - 9$ $16x^2 - 49$
Solution	(i) (ii)	$x^{2} - 9 = x^{2} - 3^{2} = (x + 3)(x - 3)$ 16x ² - 49 = (4x) ² - 7 ² = (4x + 7)(4x - 7)

Now you try these:-Factorise

1. (i) $x^{2} + 6x - 16$ (ii) $2x^{2} + 5x + 2$ (iii) $7y^{2} - 10y + 3$ (iv) $4x^{2} - 25$ (v) $16m^{2} - 81n^{2}$

Adding and subtracting algebraic fractions

Algebraic fractions follow the same rules as numerical fractions. When adding or subtracting, you need to find the common denominator, which may be a number or an algebraic expression.

Exam	ples
Simpli	fy
(i)	$\frac{2x}{3} + \frac{x}{4} - \frac{5x}{6}$
(ii)	$\frac{1}{2x} - \frac{1}{x^2}$
Solutio	on
(i)	The common denominator is 12, as 3, 4 and 6 are all factors of 12
	$\frac{2x}{3} + \frac{x}{4} - \frac{5x}{6} = \frac{8x}{12} + \frac{3x}{12} - \frac{10x}{12}$
	$=\frac{8x+3x-10x}{12}$
	$=\frac{x}{12}$
(ii)	The common denominator is $2x^2$
	$\frac{1}{2x} - \frac{1}{x^2} = \frac{x}{2x^2} - \frac{2}{2x^2}$
	$=\frac{x-2}{2x^2}$

Now you try these:-

1. Write as single fractions:

(i)
$$\frac{2x}{5} + \frac{3x}{2}$$
 (ii) $\frac{3a}{4} - \frac{2b}{3}$

(iii)
$$\frac{2x+1}{12} - \frac{x-2}{8}$$
 (iv) $\frac{3x+4}{2x} - \frac{5x+6}{3x}$

(v)
$$\frac{1}{p} + \frac{1}{q}$$
 (vi) $\frac{a}{2b} + \frac{5b}{3a}$

Simplifying fractions

You are familiar with the idea of "cancelling" to simplify numerical fractions: for example, $\frac{9}{12}$ can be simplified to $\frac{3}{4}$ by dividing both the numerator and the denominator by 3. You can also cancel before carrying out a multiplication, to make the numbers simpler: e.g. $\frac{2}{3} \times \frac{9^3}{4^2} = \frac{3}{2}$

The same technique can be used in algebra. As with factorising, remember that "cancelling" involves dividing, not subtracting.

Example

Simplify	(i)	$\frac{6xy^3 + 2x^2y}{10x^2y}$	
	(ii)	$\frac{3a}{a+1} \times \frac{2a+2}{a+2}$	

Solution

(i) It is very important to remember that you can only "cancel" if you can divide each term in both the numerator and denominator by the same expression. In this case, don't be tempted to divide by $2x^2y$ – although this is a factor of both $2x^2y$ and $10x^2y$, it is not a factor of 6xy³. In a case like this, it may be best to factorise the top first, so that it is easier to see the factors.



Now you try these:-

Simplify the following as much as possible: 1.

(i)
$$\frac{2a^2b}{4ab^2}$$
 (ii) $\frac{12p^2qr^3}{9pq^2r}$
(iii) $\frac{x^2y + xy^2}{x + y}$ (iv) $\frac{a}{2b} \times \frac{3bc}{a^2} \times \frac{a}{6c}$

(iii)
$$\frac{x^2y + xy^2}{x + y}$$
 (iv) $\frac{a}{2b} \times \frac{a}{2b}$

Linear equations

A linear equation involves only terms in x (or whatever variable is being used) and numbers. So it has no terms involving x^2 , x^3 etc. Equations like these are called linear because the graph of an expression involving only terms in x and numbers (e.g. y = 2x + 1) is always a straight line.

Solving a linear equation may involve simple algebraic techniques such as gathering like terms and multiplying out brackets.

Example

Solve these equations.		
(i) $5x - 2 = 3x + 8$		
(ii) $3(2y - 1) = 4 - 2(y - 3)$		
(iii) $\frac{2a-1}{3} = 2a+3$		
Solution		
(i) $5x - 2 = 3x + 8$	(ii)	3(2y-1) = 4 - 2(y-3)
5x = 3x + 10		6y - 3 = 4 - 2y + 6
2x = 10		6y - 3 = 10 - 2y
x = 5		8 <i>y</i> = 13
		13
		$y = \frac{1}{8}$
(iii) $\frac{2a-1}{3} = 2a+3$		
2a - 1 = 3(2a + 3)		
2a - 1 = 6a + 9		
2a = 6a + 10		
-4a = 10		
a = -2.5		

Now you try these:-

1.	Solve	e the following equations:		
	(i)	2x - 3 = 8	(ii)	3y + 2 = y - 5
	(iii)	3 - 2a = 3a - 1	(i∨)	3(p-3) = 2(2p+1)
	(~)	2(1-z) + 3(z+3) = 4z + 1	(∨i)	$\frac{2b+1}{5} = \frac{3-b}{4}$

Linear simultaneous equations

Simultaneous equations involve more than one equation and more than one unknown. To solve them you need the same number of equations as there are unknowns. One method of solving simultaneous equations involves adding or subtracting multiples of the two equations so that one unknown disappears. This method is called *elimination*, and is shown in the next example.



An alternative method of solving simultaneous equations is called substitution. This can be the easier method to use in cases where one equation gives one of the variables in terms of the other. This is shown in the next example.



- 1. Solve the following simultaneous equations:
 - (i) 2x + 5y = 11 2x - y = 5 (ii) x + 2y = 6 4x + 3y = 4 (iii) 3a - 2b = 4 5a + 4b = 3(i.i.) 2p - 5q = 5(i.i.) 5x + 3y = 9(i.i.) 3a + 2b = 1
 - (iv) 2p 3q 9 (v) 3x + 3y 7 (vi) 3a + 2b 1y = 3x - 4 9a - 4b = 4

Inequalities

Whereas the solution of an equation is a specific value, or two or more specific values, the solution of an inequality is a range of values.

Inequalities can be solved in a similar way to equations, but you do have to be very

careful, as in some situations you need to reverse the inequality. (*see example 2 below) A linear inequality involves only terms in x and constant terms.

Example

Solve the inequality 3x+1 > x-5Solution 3x+1 > x-52x+1 > -52x > -6

2x > -6x > -3



It is a good idea to check your answer by picking a number within the range of the solution and check that it satisfies the original inequality. E.g. choose a number smaller than 2, substitute into both sides of the original inequality and check the resulting statement is true.

Now you try these:-

1. Solve the following linear inequalities:

(i)
$$2x + 3 < 10$$
 (ii) $5x + 3 \ge 2x - 9$
(iii) $3x - 1 > 7 - x$ (iv) $4x + 1 \le 6x - 7$

(v) $5(x-3) \ge 2(2x+3)$ (vi) 2(1-x) > 3x+4

(vii)
$$4(2x+5) \ge 3(3x-1)$$
 (viii) $\frac{2x+1}{3} > \frac{x-4}{2}$

Surds

A surd is the square root of a whole number that has an irrational value – that is a number that cannot be written as a fraction. A surd is a number like $\sqrt{2}$, $5\sqrt{3}$ etc. (one that is written with the $\sqrt{5}$ sign.) They are important because you can give exact answers rather than rounding to a certain number of decimal places. It is important that you are able to manipulate surds as in the first year at A level although you can use calculators, there is an expectation that you complete calculations showing all non-calculator steps. You will need to know the following rules:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$a = (\sqrt{a})^2 = \sqrt{a} \times \sqrt{a}$$

Example Simplify (i)	$\sqrt{28}$	(ii)	$\sqrt{50}$		
Solution				_	
Simplifying a surd means making the number in the $\sqrt{-}$ sign smaller.					
(i)	$\sqrt{28} = \sqrt{7 \times 4}$	$=2\sqrt{7}$	(ii)	$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$	

When working in surd form, it is important to be able to manipulate expressions so that they are as simple as possible.

 Example

 Expand the brackets and write each result as simply as possible.

 (i)
 $\sqrt{3}(\sqrt{3} + 3\sqrt{5})$

 (ii)
 $(\sqrt{2} + \sqrt{3})(\sqrt{5} - \sqrt{2})$

 Solution
 (i)
 $\sqrt{3}(\sqrt{3} + 3\sqrt{5}) = 3 + 3\sqrt{15}$ (ii)
 $\sqrt{2 \times 5} - \sqrt{2 \times 2} + \sqrt{3 \times 5} - \sqrt{3 \times 2}$
 $= \sqrt{10} - \sqrt{4} + \sqrt{15} - \sqrt{6}$ $= \sqrt{10} - 2 + \sqrt{15} - \sqrt{6}$

Now you try these:-

1. (i) Express
$$\sqrt{45}$$
 in the form $k\sqrt{5}$

(ii) Write $(2+\sqrt{3})(5-2\sqrt{3})$ in the form $p+q\sqrt{3}$

SOLUTIONS

Collecting like terms

1. (i)
$$2x + 3y - x + 5y + 4x = (2x - x + 4x) + (3y + 5y)$$

= $5x + 8y$

(ii)
$$5a-2b+3c-2a+5b = (5a-2a) + (-2b+5b) + 3c$$

= $3a+3b+3c$

(iii)
$$4p+q-6p-5q+5p+4q = (4p-6p+5p)+(q-5q+4q)$$

= 3p

Multiplying out brackets

1. (i)
$$3(2x+3y) = 6x+9y$$

(ii)
$$4(3a-2b)-3(a+2b) = 12a-8b-3a-6b$$

= $9a-14b$

(iii)
$$p(2p-q) + 2q(p-3q) = 2p^2 - pq + 2qp - 6q^2$$

= $2p^2 + pq - 6q^2$

2. (i)
$$(x+1)(x-3) = x^2 - 3x + x - 3$$

= $x^2 - 2x - 3$

(ii)
$$(x+2)(2x+1) = 2x^2 + x + 4x + 2$$

= $2x^2 + 5x + 2$

(iii)
$$(x-3)(x-4) = x^2 - 4x - 3x + 12$$

= $x^2 - 7x + 12$

(iv)
$$(3x+2)(x-2) = 3x^2 - 6x + 2x - 4$$

= $3x^2 - 4x - 4$

(v)
$$(2x+1)(4x-1) = 8x^2 - 2x + 4x - 1$$

= $8x^2 + 2x - 1$

(vi)
$$(1-2x)(1+x) = 1 + x - 2x - 2x^2$$

= $1 - x - 2x^2$

(vii)
$$(3+2x)(x-1) = 3x-3+2x^2-2x$$

= $2x^2 + x - 3$
(viii) $(5x-3)(2x+5) = 10x^2 + 25x - 6x - 15$
= $10x^2 + 19x - 3$

<u>Factorising</u>

- 1. (i) 10ab + 5ac = 5a(2b + c)
 - (ii) $2x^2 + 4xy 8xz = 2x(x + 2y 4z)$
 - (iii) $3s^2t 9s^3t + 12s^2t^2 = 3s^2t(1 3s + 4t)$

Factorising quadratics

1. (i)
$$x^2 + 6x - 16 = (x+8)(x-2)$$

(ii)
$$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2 = 2x(x+2) + 1(x+2) = (2x+1)(x+2)$$

(iii)
$$7y^2 - 10y + 3 = 7y^2 - 7y - 3y + 3 = 7y(y-1) - 3(y-1) = (7y-3)(y-1)$$

(iv)
$$4x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$$

(v)
$$16m^2 - 81n^2 = (4m)^2 - (9n)^2 = (4m + 9n)(4m - 9n)$$

Adding and subtracting algebraic fractions

1. (i)
$$\frac{2x}{5} + \frac{3x}{2} = \frac{4x}{10} + \frac{15x}{10} = \frac{19x}{10}$$

(ii) $\frac{3a}{4} - \frac{2b}{3} = \frac{9a}{12} - \frac{8b}{12} = \frac{9a - 8b}{12}$
(iii) $\frac{2x + 1}{12} - \frac{x - 2}{8} = \frac{2(2x + 1)}{24} - \frac{3(x - 2)}{24} = \frac{4x + 2 - 3x + 6}{24} = \frac{x + 8}{24}$
(iv) $\frac{3x + 4}{2x} - \frac{5x + 6}{3x} = \frac{3(3x + 4)}{6x} - \frac{2(5x + 6)}{6x}$
 $= \frac{9x + 12 - 10x - 12}{6x} = \frac{-x}{6x} = -\frac{1}{6}$

$$(\vee) \qquad \frac{1}{p} + \frac{1}{q} = \frac{q}{pq} + \frac{p}{pq} = \frac{q+p}{pq}$$

(vi)
$$\frac{a}{2b} + \frac{5b}{3a} = \frac{3a^2}{6ab} + \frac{10b^2}{6ab} = \frac{3a^2 + 10b^2}{6ab}$$

Simplifying fractions

1. (i)
$$\frac{2a^2b}{4ab^2} = \frac{2 \times a \times a \times b}{4 \times a \times b \times b} = \frac{a}{2b}$$

(ii)
$$\frac{12p^2qr^3}{9pq^2r} = \frac{12 \times p \times p \times q \times r \times r \times r}{9 \times p \times q \times q \times r} = \frac{4pr^2}{3q}$$

(iii)
$$\frac{x^2y + xy^2}{x + y} = \frac{xy(x + y)}{x + y} = xy$$

(iv)
$$\frac{a}{2b} \times \frac{3bc}{a^2} \times \frac{a}{6c} = \frac{a \times 3 \times b \times c \times a}{2 \times b \times a \times a \times 6 \times c} = \frac{1}{4}$$

Linear Equations

1. (i) 2x-3=8 2x = 11 x = 5.5(ii) 3y+2=y-5 2y = -5y = -1.5

(iii)
$$3-2a = 3a-1$$
$$3 = 5a-1$$
$$4 = 5a$$
$$a = 0.8$$

(v)
$$2(1-z) + 3(z+3) = 4z + 1$$
$$2 - 2z + 3z + 9 = 4z + 1$$
$$11 + z = 4z + 1$$
$$11 = 3z + 1$$
$$10 = 3z$$
$$z = \frac{10}{3}$$

(iv)
$$3(p-3) = 2(2p+1)$$

 $3p-9 = 4p+2$
 $-9 = p+2$
 $-11 = p$
 $p = -11$
(vi) $2b+1 \quad 3-b$

(vi)
$$\frac{2b+1}{5} = \frac{5-b}{4}$$
$$4(2b+1) = 5(3-b)$$
$$8b+4 = 15 - 5b$$
$$13b+4 = 15$$
$$13b = 11$$
$$b = \frac{11}{13}$$

Linear simultaneous equations

1 2x + 5y = 11(i) 2x - y = 56y = 6y = 1Subst $2x + 5 \times 1 = 11$ 2x = 6x = 3 $\overline{2 \times 3} - 1 = 5 \quad \checkmark$ Check (ii) x + 2y = 64x + 3y = 4x4 4x + 8y = 245y = 20y = 4Subst $x + 2 \times 4 = 6$ x = -2Check in \Box 4×-2+3×4 = 4 \checkmark 3a - 2b = 4(iii) 5a + 4b = 3 $x^2 \quad 6a - 4b = 8$ 11a = 11a = 1Subst $3 \times 1 - 2b = 4$ -2b = 1 $b = -\frac{1}{2}$ Check $5 \times 1 + 4 \times -\frac{1}{2} = 3$ **✓**

(iv)
$$2p - 5q = 5$$

 $3p - 2q = -9$
x3 $6p - 15q = 15$
x2 $6p - 4q = -18$
 $-11q = 33$
 $q = -3$
Subst $2p - 5 \times -3 = 5$

$$2p = -10$$

$$p = -5$$

Check in $3 \times -5 - 2 \times -3 = -15 + 6 = -9$

 $(\vee) \qquad 5x + 3y = 9$ $\Box \ y = 3x - 4$

Substitute
$$5x + 3(3x - 4) = 9$$

 $5x + 9x - 12 = 9$
 $14x = 21$
 $x = \frac{21}{14} = \frac{3}{2}$
Subst $y = 3 \times \frac{3}{2} - 4$
 $y = \frac{9}{4} - 4 = \frac{1}{2}$
 $\frac{y = \frac{1}{2}}{\frac{15}{2}}$
Check $5 \times \frac{3}{2} + 3 \times \frac{1}{2} = \frac{15}{2} + \frac{3}{2} = 9 \checkmark$
(vi) $3a + 2b = 1$
 $9a - 4b = 4$
 x^{2} $6a + 4b = 2$
 $15a = 6$
 $a = \frac{2}{5}$
Subst $3 \times \frac{2}{5} + 2b = 1$
 $\frac{6}{5} + 2b = 1$, $2b = -\frac{1}{5}$, $\frac{b = -\frac{1}{10}}{10}$

 $9 \times \frac{2}{5} - 4 \times -\frac{1}{10} = \frac{18}{5} + \frac{2}{5} = 4$

Check

Inequalities

1. (i)
$$2x+3 < 10$$

 $2x < 7$
 $x < \frac{7}{2}$
(ii) $5x+3 \ge 2x-9$
 $3x+3 \ge -9$
 $3x \ge -12$
 $x \ge -4$
(iv) $4x+1 \le 6x-7$

(iii)
$$3x-1 > 7 - x$$

 $4x-1 > 7$
 $4x > 8$
 $x > 2$
(iv) $4x + 1 \le 6x - 7$
 $1 \le 2x - 7$
 $8 \le 2x$
 $4 \le x$
 $x \ge 4$

(v)
$$5(x-3) \le 2(2x+3)$$

 $5x-15 \le 4x+6$
 $x-15 \le 6$
 $x \le 21$

(vi)
$$2(1-x) > 3x + 4$$

 $2-2x > 3x + 4$
 $2 > 5x + 4$
 $-2 > 5x$
 $-\frac{2}{5} > x$
 $x < -\frac{2}{5}$

(vii)
$$4(2x+5) \ge 3(3x-1)$$

 $8x+20 \ge 9x-3$
 $20 \ge x-3$
 $23 \ge x$
 $x \le 23$
(viii) $\frac{2x+1}{3} > \frac{x-4}{2}$
 $2(2x+1) > 3(x-4)$
 $4x+2 > 3x-12$
 $x > -14$

<u>Surds</u>

1. (i)
$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

(ii) $(2 + \sqrt{3})(5 - 2\sqrt{3}) = 10 - 4\sqrt{3} + 5\sqrt{3} - 6$
 $= 4 + \sqrt{3}$