

## Section 2: Basic trigonometry

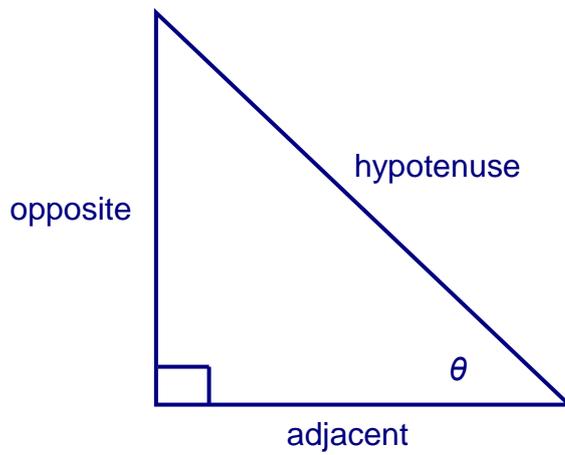
### Notes and Examples

These notes contain subsections on:

- [Basic trigonometry](#)
- [Two important triangles](#)

### Basic trigonometry

For a right-angled triangle the trigonometric functions are defined as:



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

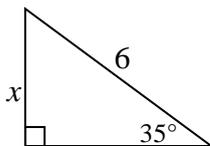
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

These relationships can be used to find sides or angles in a right-angled triangle.

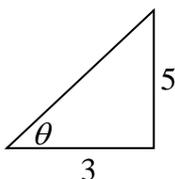


#### Example 1

- (i) Find the side marked  $x$  in the triangle below.



- (ii) Find the angle marked  $\theta$  in the triangle below.



#### Solution

- (i) The side marked  $x$  is opposite to the angle of  $35^\circ$ , and the side marked 6 is the hypotenuse, so use sine.



$$\sin 35^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{6}$$

$$x = 6 \sin 35^\circ = 3.44$$

(ii) The side marked 5 is opposite to the angle  $\theta$ , and the side marked 3 is adjacent, so use tan.

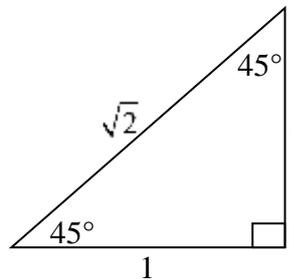
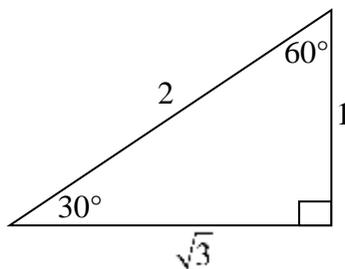
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{3}$$

$$\theta = 59.0^\circ$$

Use the inv tan or  $\tan^{-1}$  button on your calculator

### Two important triangles

The two triangles shown below are useful ones and you should remember them.



These triangles allow you to easily read off exact values for the sine, cosine and tan of the angles  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  - these angles are commonly used in problems.

From the triangles:

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\tan 45^\circ = 1$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

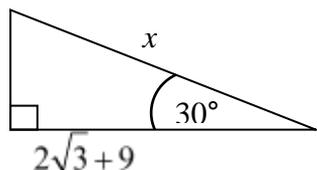
If you find it difficult to remember the triangles, you only really need to remember one fact about each one. If you remember that  $\sin 30^\circ = \frac{1}{2}$ , you can draw a triangle with angle  $30^\circ$  and mark in the opposite as 1 and the hypotenuse as 2. Then you can use Pythagoras' theorem to work out that the other side is  $\sqrt{3}$ , mark in the  $60^\circ$  angle, and you then have all the facts about  $30^\circ$  and  $60^\circ$ . Similarly, if you just remember that  $\tan 45^\circ = 1$ , you can draw a triangle with angle  $45^\circ$  and opposite and adjacent sides 1, and then you can use Pythagoras' theorem to work out that the hypotenuse is  $\sqrt{2}$ .

#### Example 2

Find the exact value of the side marked  $x$  in the triangle below.

Give your answer in the form  $p + q\sqrt{3}$  where  $p$  and  $q$  are integers





**Solution**

The side marked  $x$  is the hypotenuse of the triangle and the side marked  $2\sqrt{3} + 9$  is adjacent to the angle marked  $30^\circ$  so use cosine.

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{3} + 9}{x}$$

We know that  $\cos 30^\circ = \frac{\sqrt{3}}{2}$  so we can form an equation.

$$\begin{aligned} \frac{\sqrt{3}}{2} &= \frac{2\sqrt{3} + 9}{x} \\ x &= \frac{2(2\sqrt{3} + 9)}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{\sqrt{3}} * \frac{2(2\sqrt{3} + 9)}{\sqrt{3}} \\ &= \frac{2(6 + 9\sqrt{3})}{3} = 4 + 6\sqrt{3} \end{aligned}$$

For questions like this you would be expected to not use a calculator. You may need to review manipulating surds.

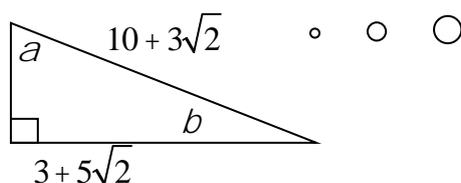


**Example 3**

The length of the hypotenuse of a right angled triangle is  $10 + 3\sqrt{2}$  and one of the legs has length  $3 + 5\sqrt{2}$ . Determine the angles in the triangle.



**Solution**



It is always worth drawing a diagram in trigonometry questions.

The hypotenuse and a leg are given so we could use either cosine to find  $b$  or sine to find  $a$ .

$$\cos b = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3 + 5\sqrt{2}}{10 + 3\sqrt{2}}$$

$$\begin{aligned} \frac{3 + 5\sqrt{2}}{10 + 3\sqrt{2}} &= \frac{3 + 5\sqrt{2}}{10 + 3\sqrt{2}} * \frac{3 - 5\sqrt{2}}{3 - 5\sqrt{2}} = \frac{9 + 15\sqrt{2} - 15\sqrt{2} - 50}{30 - 50\sqrt{2} + 9\sqrt{2} - 30} \\ &= \frac{9 - 50}{9\sqrt{2} - 50\sqrt{2}} = \frac{-41}{-41\sqrt{2}} = \frac{1}{\sqrt{2}} \end{aligned}$$

We know that  $\cos 45^\circ = \frac{1}{\sqrt{2}}$  so we can see  $b$  must be  $45^\circ$  and so  $a$  must be  $45^\circ$  too.