

Section 3: Exponentials functions

Notes and Examples

These notes contain subsections on

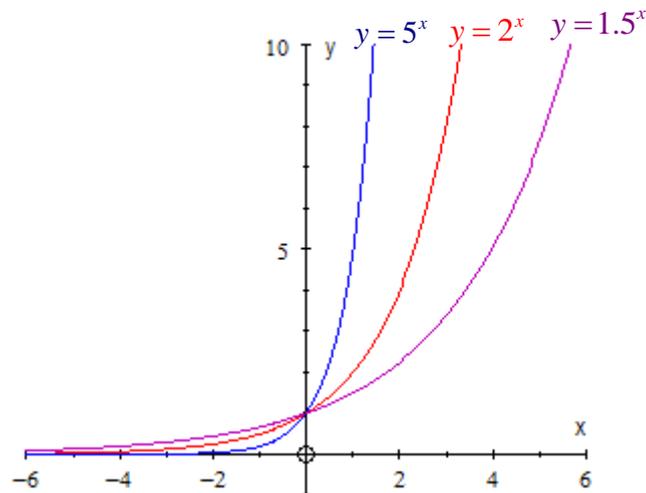
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Exponential functions

An exponential function is any function of the form $y = a^x$. The table below gives the value of y for particular values of x , for $a = 1.5, 2$ and 5 .

x	-4	-2	-1	0	1	2	3	4	5
5^x	0.0016	0.04	0.2	1	5	25	125	625	3125
2^x	0.0625	0.25	0.5	1	2	4	8	16	32
1.5^x	0.197530864	0.444444444	0.666666667	1	1.5	2.25	3.375	5.0625	7.59375

The graphs below show these exponential functions. Compare these to the values in the table above to make sure you understand the relationship between them.



Here is a simple example with an exponential function



Example 1

Let $y = 3^x$

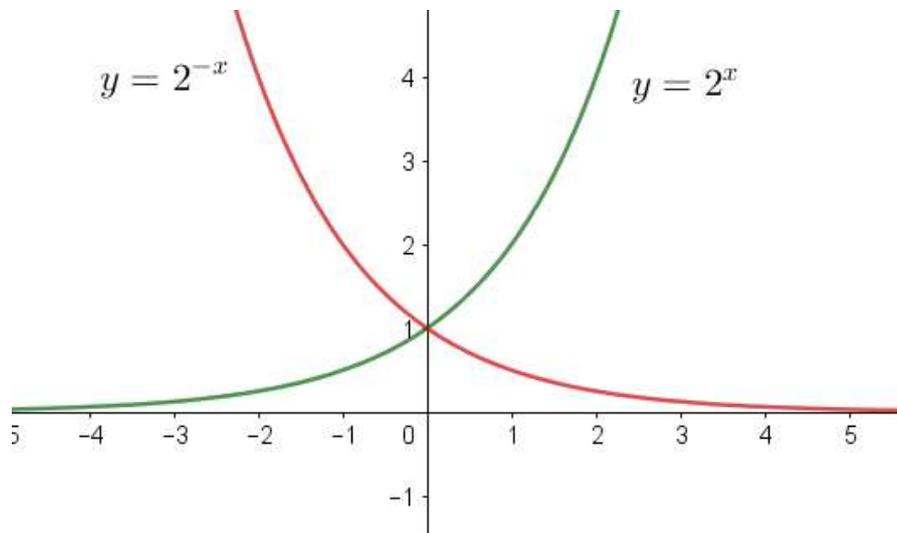
- (i) What is y when $x = -2, 1, 3$?
- (ii) For which values of x is $y = \frac{1}{3}, 1, 9$?



Solution

- (i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$, $3^1 = 3$, $3^3 = 27$ so the values of y are $\frac{1}{9}$, 3 and 27 respectively.
- (ii) $3^{-1} = \frac{1}{3}$, $3^0 = 1$, $3^2 = 9$ so the values of x are -1, 0 and 2 respectively.

In the diagram below you see the graphs $y = 2^x$ and $y = 2^{-x}$. Note that each is a reflection of the other.



$y = 2^x$ increases as x increases whereas $y = 2^{-x}$ decreases. This is true of $y = a^x$ and $y = a^{-x}$ where a is positive real number. You may wish to investigate this using graphing software with sliders.

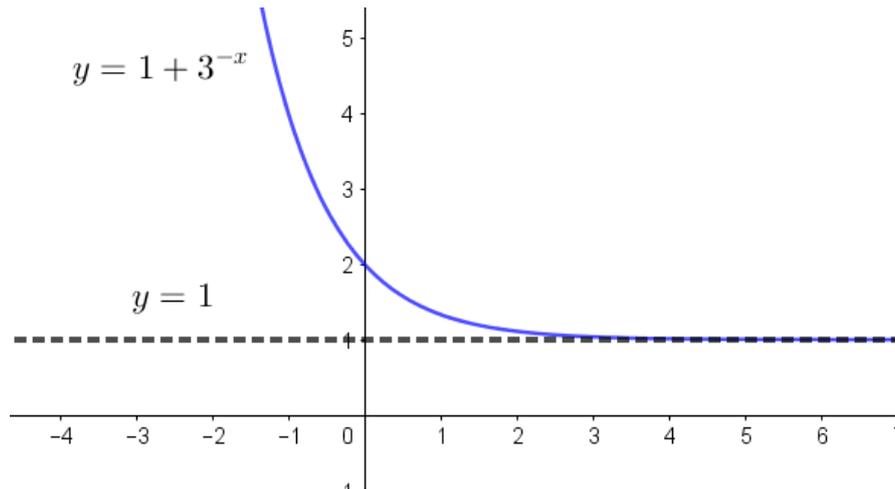
Applications of Exponential functions

Many real-life situations can be modelled by exponential functions. The growth of a population (e.g. of people, animals or bacteria) can be modelled by an exponential function. A model like this might take the form $y = c \times a^{kt}$. This type of model is called **exponential growth**.

In an exponential growth model, the quantity being modelled continues to increase, at an ever-increasing rate. In a real-life situation such as the growth of a population, the model will eventually break down, since other factors such as overcrowding or limited resources will affect the growth of the population.

Another type of model is **exponential decay**, in which something decreases exponentially. A model like this might take the form $y = c \times a^{-kt}$. Exponential decay could model the temperature of a cooling liquid, or the mass of a radioactive isotope remaining.

In an exponential decay model, the quantity being modelled decreases at a rate which becomes slower and slower. The quantity will approach a limiting value, but never quite reach it. For example, the graph below shows the curve $y = 1 + 3^{-x}$. The graph approaches the line $y = 1$ as x becomes large.



Example 2

A cup of coffee that has just been made at time $t = 0$, cools as modelled by the formula below, where T in $^{\circ}\text{C}$ is the temperature of the coffee at time t minutes.

$$T = 18 + 54 \times 2.7^{-t/27}$$

What is the temperature of the coffee

- (i) when it has just been made?
- (ii) one minute after it has been made?
- (iii) one hour after it has been made?



Solution

(i) At time $t = 0$ mins, the temperature of the coffee is $T = 18 + 54 \times 2.7^0 = 72^{\circ}\text{C}$

(ii) At time $t = 1$ mins, the temperature of the coffee is

$$T = 18 + 54 \times 2.7^{-1/27} = 70.049591^{\circ}\text{C}.$$

(iii) At time $t = 60$ mins, the temperature of the coffee is

$$T = 18 + 54 \times 2.7^{-60/27} = 23.940290^{\circ}\text{C}.$$