# AQA Level 2 Further Mathematics Number & Algebra I

## Section 3: Surds

### Solutions to Exercise

1. (i) $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

   (ii) $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

   (iii) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

   (iv) $\sqrt{216} = \sqrt{36 \times 6} = \sqrt{36} \times \sqrt{6} = 6\sqrt{6}$

   (v) $\sqrt{63} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$

   (vi) $\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

   (vii) $\sqrt{6} \times \sqrt{27} = \sqrt{3} \times \sqrt{2} \times \sqrt{9} \times \sqrt{3} = \sqrt{2} \times 3 \times 3 = 9\sqrt{2}$

   (viii) $\sqrt{12} \times \sqrt{15} = \sqrt{4} \times \sqrt{3} \times \sqrt{3} \times \sqrt{5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}$

   (ix) $\sqrt{10} \times \sqrt{24} \times \sqrt{15} = \sqrt{5} \times \sqrt{2} \times \sqrt{4} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3} \times \sqrt{5}$
   $$= 5 \times 2 \times 2 \times 3$$
   $$= 60$$

2. (i) $\left(1 + \sqrt{2}\right) + \left(3 - 2\sqrt{2}\right) = 1 + 3 + \sqrt{2} - 2\sqrt{2}$
   $$= 4 - \sqrt{2}$$

   (ii) $\left(5\sqrt{2} - 2\sqrt{3}\right) - \left(\sqrt{2} + 3\sqrt{3}\right) = 5\sqrt{2} - 2\sqrt{3} - \sqrt{2} - 3\sqrt{3}$
   $$= 4\sqrt{2} - 5\sqrt{3}$$

   (iii) $2\left(\sqrt{5} - 3\sqrt{3}\right) + 3\left(2\sqrt{5} + \sqrt{3}\right) = 2\sqrt{5} - 6\sqrt{3} + 6\sqrt{5} + 3\sqrt{3}$
   $$= 8\sqrt{5} - 3\sqrt{3}$$

   (iv) $\sqrt{18} + \sqrt{72} - \sqrt{98} = \sqrt{9 \times 2} + \sqrt{36 \times 2} - \sqrt{49 \times 2}$
   $$= 3\sqrt{2} + 6\sqrt{2} - 7\sqrt{2}$$
   $$= 2\sqrt{2}$$

# AQA FM Number and Algebra 1 Exercise solutions

3. (i) $\left(1+\sqrt{2}\right)\left(3-\sqrt{2}\right) = 3 - \sqrt{2} + 3\sqrt{2} - 2$

$$= 1 + 2\sqrt{2}$$

(ii) $\left(2-\sqrt{3}\right)\left(3+2\sqrt{3}\right) = 6 + 4\sqrt{3} - 3\sqrt{3} - 2 \times 3$

$$= \sqrt{3}$$

(iii) $\left(3-2\sqrt{5}\right)\left(1-3\sqrt{5}\right) = 3 - 9\sqrt{5} - 2\sqrt{5} + 6 \times 5$

$$= 33 - 11\sqrt{5}$$

(iv) $\left(\sqrt{2}+2\sqrt{3}\right)\left(5\sqrt{2}-\sqrt{3}\right) = 5 \times 2 - \sqrt{2}\sqrt{3} + 10\sqrt{3}\sqrt{2} - 2 \times 3$

$$= 10 - \sqrt{6} + 10\sqrt{6} - 6$$

$$= 4 + 9\sqrt{6}$$

(v) $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right) = 7 - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - 2$

$$= 5$$

(vi) $\left(3-\sqrt{2}\right)^2 = \left(3-\sqrt{2}\right)\left(3-\sqrt{2}\right)$

$$= 9 - 3\sqrt{2} - 3\sqrt{2} + 2$$

$$= 11 - 6\sqrt{2}$$

4. (i) $\dfrac{3}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

(ii) $\dfrac{1}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

(iii) $\dfrac{1+\sqrt{2}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\left(1+\sqrt{2}\right)\sqrt{2}}{2} = \dfrac{\sqrt{2}+2}{2}$

(iv) $\dfrac{1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{\sqrt{3}-1}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \dfrac{\sqrt{3}-1}{3-1} = \dfrac{\sqrt{3}-1}{2}$

(v) $\dfrac{\sqrt{2}}{2-\sqrt{2}} \times \dfrac{2+\sqrt{2}}{2+\sqrt{2}} = \dfrac{\sqrt{2}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)} = \dfrac{2\sqrt{2}+2}{4-2} = \dfrac{2\sqrt{2}+2}{2} = \sqrt{2}+1$

(vi) $\dfrac{1-\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}} = \dfrac{\left(1-\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$

$$= \dfrac{2+\sqrt{3}-2\sqrt{3}-3}{4-3}$$

$$= \dfrac{-1-\sqrt{3}}{1}$$

$$= -1-\sqrt{3}$$

(vii) $\dfrac{1+2\sqrt{5}}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{\left(1+2\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$

$$= \dfrac{3+\sqrt{5}+6\sqrt{5}+2\times 5}{9-5}$$

$$= \dfrac{13+7\sqrt{5}}{4}$$

(viii) $\dfrac{1+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{\left(1+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$

$$= \dfrac{\sqrt{3}-\sqrt{2}+\sqrt{6}-2}{3-2}$$

$$= \sqrt{3}-\sqrt{2}+\sqrt{6}-2$$

(ix) $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}} \times \dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \dfrac{\left(\sqrt{6}+\sqrt{3}\right)\left(\sqrt{6}+\sqrt{3}\right)}{\left(\sqrt{6}-\sqrt{3}\right)\left(\sqrt{6}+\sqrt{3}\right)}$

$$= \dfrac{6+\sqrt{18}+\sqrt{18}+3}{6-3}$$

$$= \dfrac{9+2\sqrt{18}}{3}$$

5.  One possible width is $\left(\sqrt{7}-1\right)$ cm.

In this case, the area is $\left(\sqrt{7}+1\right)\left(\sqrt{7}-1\right) = 7-1 = 6$ cm².

Other answers are possible.