

Section 5: Indices

Solutions to Exercise

1. (i) $3^4 = 3 \times 3 \times 3 \times 3 = 81$
(ii) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
(iii) $4^{1/2} = \sqrt{4} = 2$
(iv) $16^{-1/2} = \frac{1}{\sqrt{16}} = \frac{1}{4}$
(v) $8^{5/3} = (\sqrt[3]{8})^5 = 2^5 = 32$
(vi) $36^{-3/2} = \frac{1}{(\sqrt{36})^3} = \frac{1}{6^3} = \frac{1}{216}$
(vii) $\left(\frac{1}{2}\right)^{-1} = (2^{-1})^{-1} = 2^1 = 2$
(viii) $\left(\frac{25}{9}\right)^{-1/2} = \left(\frac{9}{25}\right)^{1/2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$
(ix) $\left(\frac{27}{64}\right)^{-2/3} = \left(\frac{64}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{64}{27}}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$
2. (i) $3^{11} \times 3^{-4} \div 3^3 = 3^{11-4-3} = 3^4 = 81$
(ii) $(2^5)^3 \times (2^7)^{-2} = 2^{15} \times 2^{-14} = 2^{15-14} = 2^1 = 2$
(iii) $\frac{5^6}{5^5 \times 5^3} = 5^{6-5-3} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
3. (i) $2^3 \times 16^{\frac{1}{2}} = 2^3 \times (2^4)^{\frac{1}{2}}$
 $= 2^3 \times 2^2$
 $= 2^5 (= 32)$
- (ii) $\frac{3^5 \times 5^3}{\sqrt{81 \times 25}} = \frac{3^5 \times 5^3}{\sqrt{3^4 \times 5^2}}$
 $= \frac{3^5 \times 5^3}{3^2 \times 5}$
 $= 3^3 \times 5^2 (= 675)$

$$4. \quad (i) \quad \frac{2^5 \times 4^{1/2}}{2} = \frac{2^5 \times (2^2)^{1/2}}{2} = \frac{2^5 \times 2^1}{2} = 2^{5+1-1} = 2^5 = 32$$

$$(ii) \quad (3^5)^{3/2} \times 9^{-7/4} = (3^5)^{3/2} \times (3^2)^{-7/4} = 3^{15/2} \times 3^{-7/2} = 3^{15/2 - 7/2} = 3^4 = 81$$

$$(iii) \quad \sqrt{\frac{x^{4/3}}{x^{1/3} \times x^{8/3}}} = \sqrt{x^{\frac{4}{3} - \frac{1}{3} - \frac{8}{3}}} = \sqrt{x^{-\frac{5}{3}}} = (x^{-\frac{5}{3}})^{\frac{1}{2}} = x^{-\frac{5}{6}}$$

$$5. \quad (i) \quad \frac{16x^{\frac{1}{2}}}{2^3 x^{-\frac{1}{2}}} = \frac{2^4 x^{\frac{1}{2}}}{2^3 x^{-\frac{1}{2}}} = 2x$$

$$(ii) \quad \frac{x^{\frac{5}{4}} \cdot x^{-1}}{\sqrt[4]{x^3}} = \frac{x^{\frac{1}{4}}}{x^{\frac{3}{4}}} = x^{-\frac{1}{2}}$$

$$6. \quad (i) \quad 3^{5/2} - 3^{1/2} = 3^{1/2} \times 3^2 - 3^{1/2} = 3^{1/2}(3^2 - 1) = \sqrt{3} \times 8 = 8\sqrt{3}$$

$$(ii) \quad 2^{1/2} + 2^{3/2} + 2^{5/2} = 2^{1/2} + 2^{1/2} \times 2^1 + 2^{1/2} \times 2^2 = 2^{1/2}(1 + 2 + 2^2) = 7\sqrt{2}$$

$$(iii) \quad y^{1/2} - y^{-1/2} = \sqrt{y} - \frac{1}{\sqrt{y}} = \frac{y-1}{\sqrt{y}}$$

$$7. \quad (i) \quad \frac{2^{\frac{5}{2}} - 2^{\frac{3}{2}}}{2^{\frac{1}{2}}} = \frac{2^{\frac{3}{2}}(2-1)}{2^{\frac{1}{2}}} = 2$$

$$(ii) \quad \left(\frac{x^{\frac{7}{4}} - x^{\frac{3}{4}} + x \cdot x^{\frac{7}{4}}}{x^{\frac{1}{4}}} \right)^2 = \left(\frac{x^{\frac{3}{4}} [x-1+x^2]}{x^{\frac{1}{4}}} \right)^2 = (x^{\frac{1}{2}} [x^2+x-1])^2 = x(x^2+x-1)^2$$

$$\begin{aligned}
 \text{(iii)} \quad \left[\frac{y^{\frac{1}{2}}}{x^{\frac{3}{4}}} - \frac{x^{\frac{5}{4}}}{y^{\frac{3}{2}}} \right]^4 &= \left[\frac{y^{\frac{1}{2}} \cdot y^{\frac{3}{2}} - x^{\frac{5}{4}} \cdot x^{\frac{3}{4}}}{x^{\frac{3}{4}} \cdot y^{\frac{3}{2}}} \right]^4 \\
 &= \left[\frac{y^2 - x^2}{x^{\frac{3}{4}} \cdot y^{\frac{3}{2}}} \right]^4 \\
 &= \frac{(y^2 - x^2)^4}{x^3 \cdot y^6}
 \end{aligned}$$

8.

(i)

$$\begin{aligned}
 x - 8\sqrt{x} + 15 &= 0 \\
 (\sqrt{x} - 3)(\sqrt{x} - 5) &= 0 \\
 \sqrt{x} = 3 \text{ or } \sqrt{x} = 5 \\
 x = 3^2 = 9 \text{ or } x = 5^2 = 25
 \end{aligned}$$

(ii)

$$\begin{aligned}
 \frac{1}{x^2} - \frac{1}{x} - 2 &= 0 \text{ multiply by } x^2 \\
 1 - x - 2x^2 &= 0 \\
 2x^2 + x - 1 &= 0 \\
 (2x - 1)(x + 1) &= 0 \\
 2x = 1 \text{ or } x = -1 \\
 x = \frac{1}{2} \text{ or } x = -1
 \end{aligned}$$

9.

(i) $2^x = \frac{1}{4}$

The power of 2 that gives $\frac{1}{4}$ is -2, i.e. $2^{-2} = \frac{1}{4}$ so, the solution is $x = -2$

(ii)

$$\begin{aligned}
 \left(\frac{1}{3}\right)^x &= 27 \\
 \left(\frac{1}{3}\right)^x &= (3^{-1})^x \text{ and } 27 = 3^3 \\
 \text{So, } (3^{-1})^x &= 3^3 \\
 \text{So, } x &= -3
 \end{aligned}$$

(iii)

$$\frac{1}{5} = 25^x$$

$$25^x = (5^2)^x = 5^{2x} \text{ and } \frac{1}{5} = 5^{-1}$$

$$\text{So, } 5^{2x} = 5^{-1}$$

$$\text{So, } 2x = -1$$

$$\text{So, } x = -\frac{1}{2}$$