

## Section 1: Straight lines

### Notes and Examples

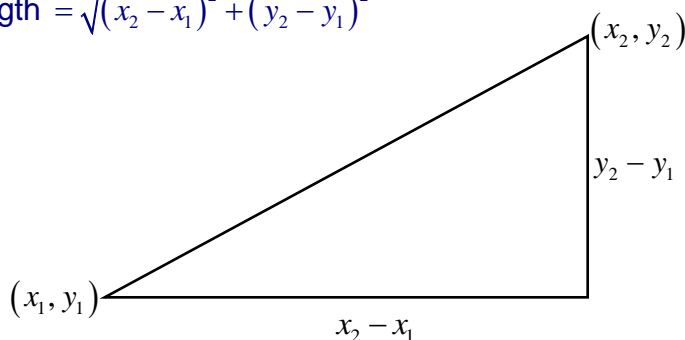
These notes contain sub-sections on:

- [Distance between two points](#)
- [Midpoints and other points of intersection](#)
- [Parallel and perpendicular lines](#)

### Distance between two points

The length of a line joining two points  $(x_1, y_1)$  and  $(x_2, y_2)$  can be found using Pythagoras' Theorem.

$$\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



#### Example 1

A is the point  $(2, -6)$ . B is the point  $(-3, 4)$ . Find the length of AB.

#### Solution

The distance AB is given by

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(2 - (-3))^2 + ((-6) - 4)^2} \\ &= \sqrt{(5)^2 + (-10)^2} \\ &= \sqrt{25 + 100} \\ &= \sqrt{125} \end{aligned}$$

Note: The answer is often left like this if the square root is not exact. However since  $125 = 25 \times 5$  then  $\sqrt{125} = \sqrt{25} \sqrt{5} = 5\sqrt{5}$  is perhaps a simpler form.



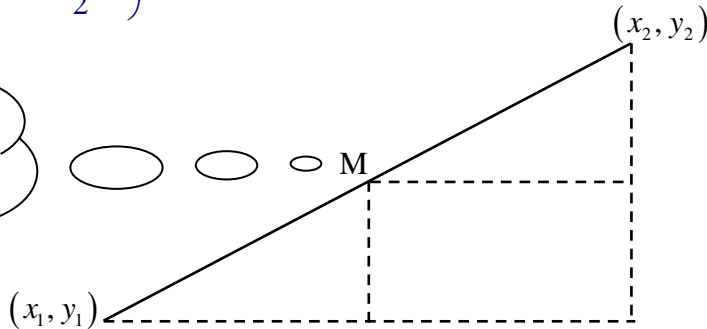
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## Midpoints and other points of intersection

The midpoint of a line joining two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

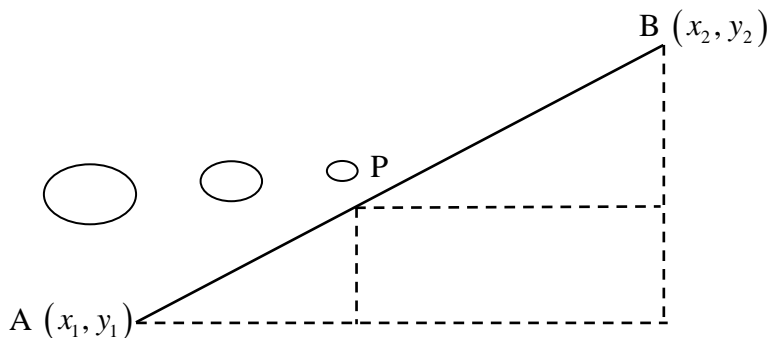
$$\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The  $x$ -coordinate of M is halfway between  $x_1$  and  $x_2$ .  
The  $y$ -coordinate of M is halfway between  $y_1$  and  $y_2$ .



Similarly, you can find the coordinates of any point on a line which divides the line in a given ratio. For example, the diagram below shows a point P which divides the line AB in the ratio 2:3.

The  $x$ -coordinate of P is  $\frac{2}{5}$  of the way between  $x_1$  and  $x_2$ . The  $y$ -coordinate of P is  $\frac{2}{5}$  of the way between  $y_1$  and  $y_2$ .



So, the coordinates of P are  $(x_1 + \frac{2}{5}(x_2 - x_1), y_1 + \frac{2}{5}(y_2 - y_1))$



### Example 2

A is the point (2, -6). B is the point (-3, 4).

- (i) Find the midpoint of AB
- (ii) The point C divides the line AB in the ratio 3:1. Find the coordinates of C.

Choose A as  $(x_1, y_1)$  and B as  $(x_2, y_2)$ .

or vice versa, it will still give the same answer (**WHY?**)

### Solution

- (i) Midpoint is  $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$   
i.e.  $\left( \frac{2 + (-3)}{2}, \frac{-6 + 4}{2} \right)$



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$$= \left( \frac{-1}{2}, -1 \right)$$

- (ii) The distance between A and B in the  $x$ -direction is -5.  
The  $x$ -coordinate of C is  $2 + \frac{3}{4} \times -5 = 2 - 3.75 = -1.75$   
The distance between A and B in the  $y$ -direction is 10.  
The  $y$ -coordinate of C is  $-6 + \frac{3}{4} \times 10 = -6 + 7.5 = 1.5$   
C is the point  $(-1.75, 1.5)$ .

## Parallel and perpendicular lines

If two lines are parallel, they have the same gradient.

If two lines with gradients  $m_1$  and  $m_2$  are perpendicular, then  $m_1 m_2 = -1$



### Example 3

P is the point  $(-3, 7)$ . Q is the point  $(5, 1)$ .

Calculate

- (i) the gradient of PQ
- (ii) the gradient of a line parallel to PQ
- (iii) the gradient of a line perpendicular to PQ.

### Solution

- (i) Choose P as  $(x_1, y_1)$  and Q as  $(x_2, y_2)$ .

$$\text{Gradient of PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{5 - (-3)} = \frac{-6}{8} = -\frac{3}{4}$$

or vice versa: it will still give the same answer (**WHY?**)

Notes:

- (1) Draw a sketch and check that your answer is sensible (e.g. has negative gradient).
- (2) Check that you get the same result when you choose Q as  $(x_1, y_1)$  and P as  $(x_2, y_2)$ .

- (ii) When two lines are parallel their gradients are equal. ( $m_1 = m_2$ )

So the gradient of the line parallel to PQ is also  $-\frac{3}{4}$ .

- (iii) When two lines are perpendicular  $m_1 m_2 = -1$ .

$$\text{So } -\frac{3}{4} m_2 = -1$$

$$\Rightarrow m_2 = \frac{4}{3}$$



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The gradient of a line perpendicular to PQ is  $\frac{4}{3}$ .



## Example 4

A straight line L has equation  $y = 2x - 5$ .

- (i) Find the equation of the line parallel to L and passing through (3, -1).
- (ii) Find the equation of the line perpendicular to L and passing through (3, -1).

## Solution

Line L has gradient 2.

- (i) Any line parallel to L has gradient 2.

The equation of the line is  $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-1) = 2(x - 3)$$

$$\Rightarrow y + 1 = 2x - 6$$

$$\Rightarrow y = 2x - 7$$

$m = 2$  and  
 $(x_1, y_1)$  is (3, -1)

You should check that the point (3, -1) satisfies your line. If it doesn't, you must have made a mistake!

- (ii) For two perpendicular lines  $m_1 m_2 = -1$ , so the gradient of the new line is  $-\frac{1}{2}$ .

The equation of the line is  $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-1) = -\frac{1}{2}(x - 3)$$

$$\Rightarrow -2y - 2 = x - 3$$

$$\Rightarrow -2y = x - 1$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2}$$

$m = -\frac{1}{2}$  and  
 $(x_1, y_1)$  is (3, -1)

The final form of the equation can be written in various different ways:

e.g.  $2y = -x + 1$  (This form has no fractions.)

e.g.  $2y + x = 1$  (This has no fractions and avoids having a negative sign at the start of the right hand side.)

The **perpendicular bisector** of a line joining two points A and B is the line that is perpendicular to AB and passes through the midpoint of AB (i.e. it **bisects** AB).



## Example 5

Find the equation of the perpendicular bisector of the points A (3, 2) and B (-1, 8).

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### Solution

$$\text{Gradient of AB} = \frac{8-2}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$$

$$\text{Gradient of perpendicular line} = \frac{2}{3}$$

$$\text{Midpoint of AB is } \left( \frac{3+(-1)}{2}, \frac{2+8}{2} \right) = (1, 5)$$

The equation of the line is  $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 5 = \frac{2}{3}(x - 1)$$

$$\Rightarrow 3y - 15 = 2x - 2$$

$$\Rightarrow 3y = 2x + 13$$