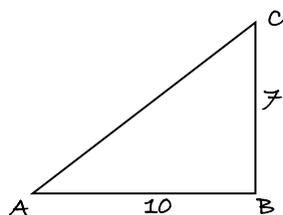


Section 2: Lines and planes in 3D

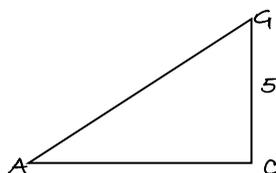
Solutions to Exercise

1. (i)



$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 10^2 + 7^2 \\ &= 149 \end{aligned}$$

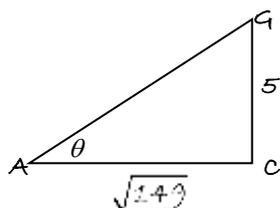
$$AC = \sqrt{149} = 12.2 \text{ (3 s.f.)}$$



$$\begin{aligned} AG^2 &= AC^2 + CG^2 \\ &= 149 + 5^2 \\ &= 174 \end{aligned}$$

$$AG = \sqrt{174} = 13.2 \text{ (3 s.f.)}$$

(ii) The angle between AG and the plane ABCD is the angle GAC.



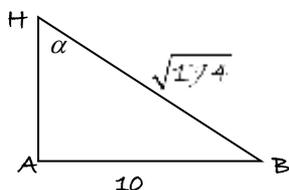
$$\tan \theta = \frac{5}{\sqrt{149}}$$

$$\theta = 22.3^\circ$$

The angle between AG and the plane ABCD is 22.3° (3 s.f.)

(iii) The angle between BH and the plane ADHE is the angle BHA.

The length of BH is the same as the length of AG.

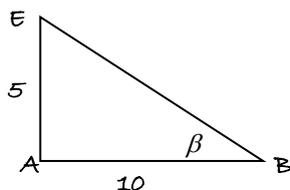


$$\sin \alpha = \frac{10}{\sqrt{174}}$$

$$\alpha = 49.3^\circ$$

The angle between BH and the plane ADHE is 49.3° (3 s.f.)

(iv) The angle between the plane BCHE and the plane ABCD is the angle EBA.

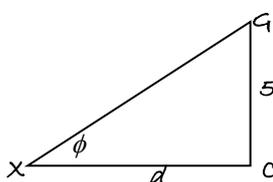
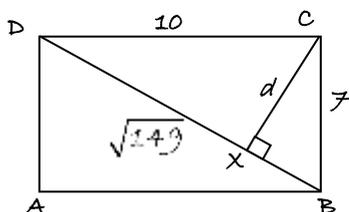


$$\tan \beta = \frac{5}{10}$$

$$\beta = 26.6^\circ$$

The angle between the planes BCHE and ABCD is 26.6° (3 s.f.)

- (v) The angle between the plane BDG and the plane ABCD is the angle GXC, where X is the point on DB such that CX is perpendicular to DB.



Triangle BCD is similar to triangle BXC, so $\frac{d}{7} = \frac{10}{\sqrt{149}}$

$$d = \frac{70}{\sqrt{149}}$$

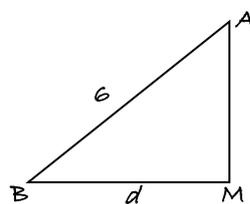
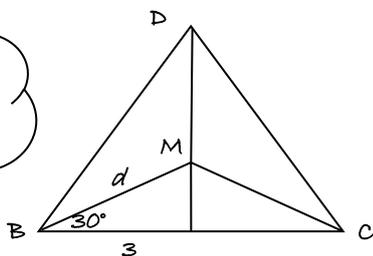
$$\tan \phi = \frac{5}{d} = \frac{5\sqrt{149}}{70}$$

$$\phi = 41.1^\circ$$

The angle between the planes BDG and ABCD is 41.1° (3 s.f.).

2. (i) Let M be the point on the base directly below the vertex A.

Use the exact value of $\cos 30^\circ$ from the standard triangle



$$\cos 30^\circ = \frac{3}{d}$$

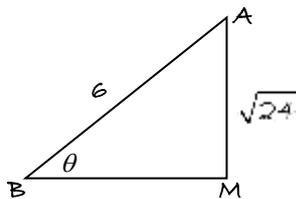
$$d = \frac{3}{\cos 30^\circ} = \frac{3}{\frac{1}{2}\sqrt{3}} = 2\sqrt{3}$$

$$AM^2 = 6^2 - d^2$$

$$= 36 - 12$$

$$AM = \sqrt{24} = 4.90 \text{ (3 s.f.)}$$

(ii) The angle between the line AB and the plane BCD is the angle ABM.

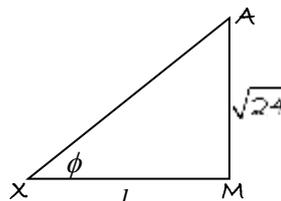
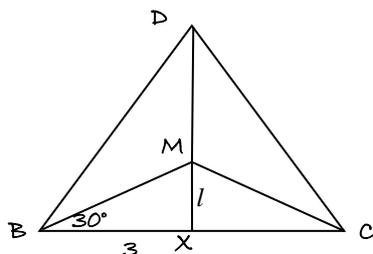


$$\sin \theta = \frac{\sqrt{24}}{6}$$

$$\theta = 54.7^\circ$$

The angle between AB and the plane BCD is 54.7° (3 s.f.)

(iii) The angle between the plane ABC and the plane BCD is the angle AXM, where X is the midpoint of BC.



$$\tan 30^\circ = \frac{l}{3}$$

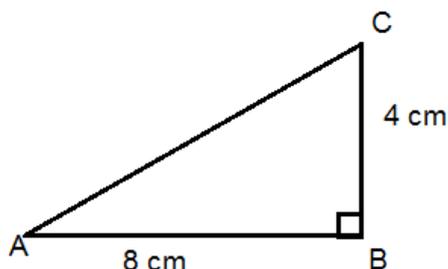
$$l = 3 \tan 30^\circ = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\tan \phi = \frac{\sqrt{24}}{\sqrt{3}}$$

$$\phi = 70.5^\circ$$

The angle between the planes is 70.5° (3 s.f.)

3.



$$AC^2 = AB^2 + BC^2$$

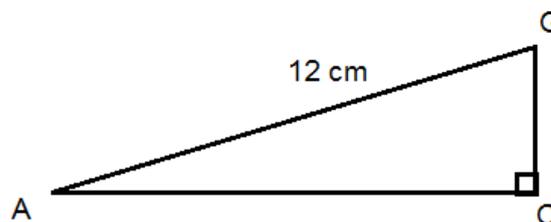
$$AC^2 = 64 + 16 = 80$$

$$AG^2 = AC^2 + GC^2$$

$$144 = 80 + GC^2$$

$$GC^2 = 64$$

$$GC = 8 \text{ cm.}$$



Volume = $8 \times 8 \times 4 = 256 \text{ cm}^3$.

4.

(i)

$$FH^2 = 12^2 + 12^2$$

$$FH = \sqrt{288} = 12\sqrt{2}$$

$$FM^2 = 12^2 + 35^2$$

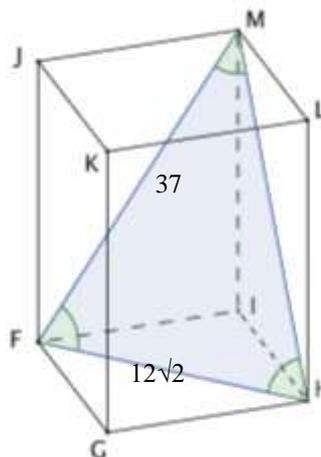
$$FM = \sqrt{1369} = 37$$

$$FM = HM = 37$$

$$\cos(\angle HFM) = \frac{37^2 + (12\sqrt{2})^2 - 37^2}{2 * 37 * 12\sqrt{2}}$$

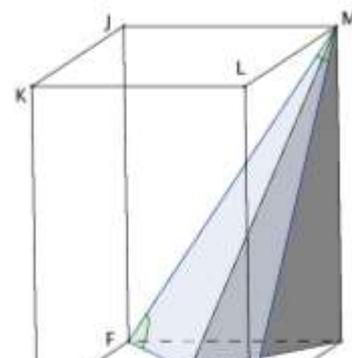
$$\cos(\angle HFM) = \frac{288}{888\sqrt{2}}$$

$$\angle HFM = \cos^{-1}\left(\frac{288}{888\sqrt{2}}\right) = 76.7^\circ$$



(ii) the line of greatest slope will go through the mid point of FH, call this point O.

$$\angle MOI = \tan^{-1}\left(\frac{37}{6\sqrt{2}}\right) = 77.1^\circ$$



5. (i) $\angle DAB = \angle DBA = \frac{180 - 30}{2} = 75^\circ$

$$\frac{2}{\sin 30} = \frac{AD}{\sin 75}$$

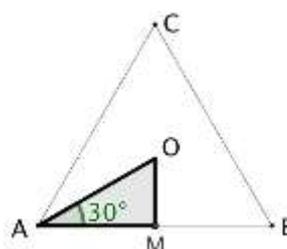
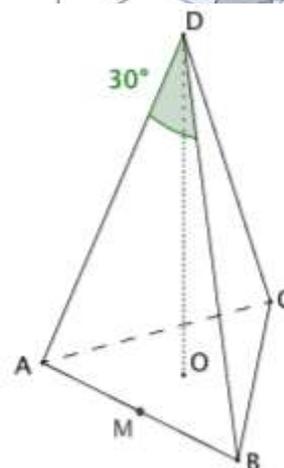
$$AD = \frac{2 \sin 75}{1/2} = 4 \sin 75 = 3.86$$

(ii)

$$DM^2 = AD^2 - AM^2$$

$$DM = \sqrt{(4 \sin 75)^2 - 1^2} = 3.73$$

$$\tan 30 = \frac{OM}{1} = \frac{1}{\sqrt{3}}$$



$$DO^2 = DM^2 - OM^2$$

$$DO = \sqrt{(4 \sin 75)^2 - 1 - \frac{1}{3}} = 3.69$$

(iii)

Angle between $\triangle ABD$ and $\triangle ABC$ is angle DMO .

$$\cos(DMO) = \frac{OM}{DM}$$

$$\cos(DMO) = \frac{1/\sqrt{3}}{\sqrt{(4 \sin 75)^2 - 1}}$$

$$DMO = \cos^{-1} \left(\frac{1}{\sqrt{3((4 \sin 75)^2 - 1)}} \right)$$

$$= 81.1^\circ$$

6.

(i) $\angle AOB = 180 - 100 - 70 = 10$

$$\frac{20}{\sin(10)} = \frac{OA}{\sin(100)}$$

$$OA = \frac{20 \sin(100)}{\sin(10)} = 113m$$

(ii) $OH = 113 \tan(10) = 20m$

