

Section 1: Straight lines

Notes and Examples

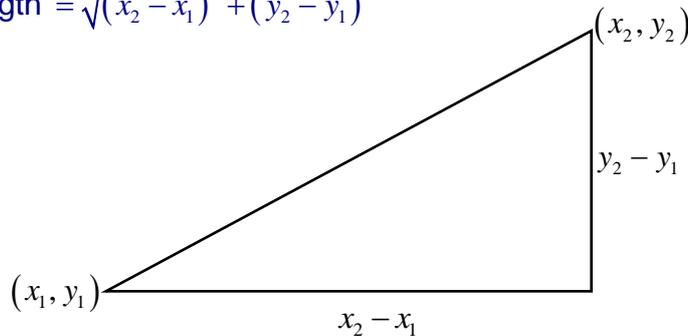
These notes contain sub-sections on:

- [Distance between two points](#)
- [Midpoints and other points of intersection](#)
- [Parallel and perpendicular lines](#)

Distance between two points

The length of a line joining two points (x_1, y_1) and (x_2, y_2) can be found using Pythagoras' Theorem.

$$\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1

A is the point $(2, -6)$. B is the point $(-3, 4)$. Find the length of AB.

Solution

The distance AB is given by

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(2 - (-3))^2 + ((-6) - 4)^2} \\ &= \sqrt{(5)^2 + (-10)^2} \\ &= \sqrt{25 + 100} \\ &= \sqrt{125} \end{aligned}$$

Note: The answer is often left like this if the square root is not exact. However since $125 = 25 \times 5$ then $\sqrt{125} = \sqrt{25} \sqrt{5} = 5\sqrt{5}$ is perhaps a simpler form.

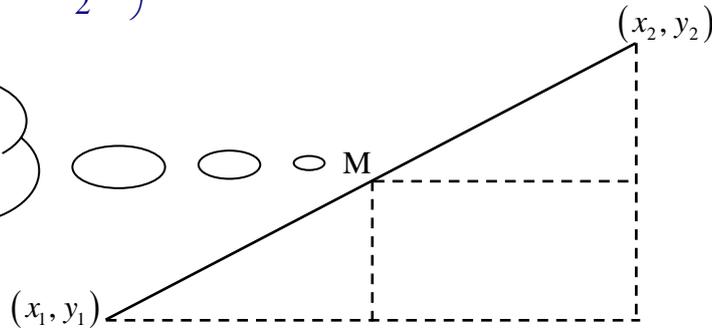
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Midpoints and other points of intersection

The midpoint of a line joining two points (x_1, y_1) and (x_2, y_2) is given by

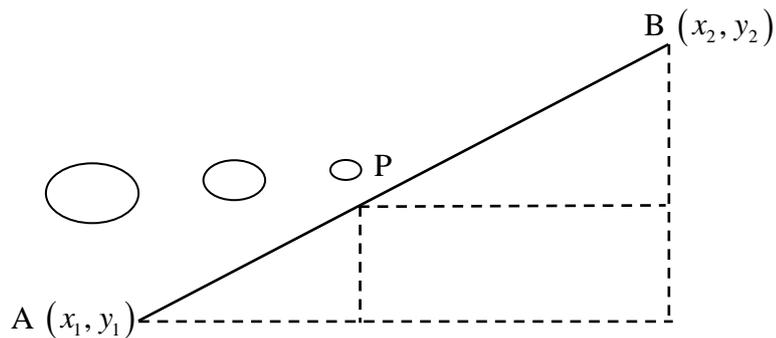
$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The x -coordinate of M is halfway between x_1 and x_2 .
The y -coordinate of M is halfway between y_1 and y_2 .



Similarly, you can find the coordinates of any point on a line which divides the line in a given ratio. For example, the diagram below shows a point P which divides the line AB in the ratio 2:3.

The x -coordinate of P is $\frac{2}{5}$ of the way between x_1 and x_2 . The y -coordinate of P is $\frac{2}{5}$ of the way between y_1 and y_2 .



So, the coordinates of P are $(x_1 + \frac{2}{5}(x_2 - x_1), y_1 + \frac{2}{5}(y_2 - y_1))$



Example 2

A is the point (2, -6). B is the point (-3, 4).

- Find the midpoint of AB
- The point C divides the line AB in the ratio 3:1. Find the coordinates of C.

Choose A as (x_1, y_1) and B as (x_2, y_2) .

or vice versa, it will still give the same answer (**WHY?**)

Solution

- Midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
i.e. $\left(\frac{2 + (-3)}{2}, \frac{-6 + 4}{2} \right)$



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$$= \left(\frac{-1}{2}, -1 \right)$$

- (ii) The distance between A and B in the x -direction is -5 .
The x -coordinate of C is $2 + \frac{3}{4} \times -5 = 2 - 3.75 = -1.75$
The distance between A and B in the y -direction is 10 .
The y -coordinate of C is $-6 + \frac{3}{4} \times 10 = -6 + 7.5 = 1.5$
C is the point $(-1.75, 1.5)$.

Parallel and perpendicular lines

If two lines are parallel, they have the same gradient.

If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 m_2 = -1$



Example 3

P is the point $(-3, 7)$. Q is the point $(5, 1)$.

Calculate

- the gradient of PQ
- the gradient of a line parallel to PQ
- the gradient of a line perpendicular to PQ.

Solution

- (i) Choose P as (x_1, y_1) and Q as (x_2, y_2) .

$$\text{Gradient of PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{5 - (-3)} = \frac{-6}{8} = -\frac{3}{4}$$

or vice versa: it will still give the same answer (**WHY?**)

Notes:

- Draw a sketch and check that your answer is sensible (e.g. has negative gradient).
- Check that you get the same result when you choose Q as (x_1, y_1) and P as (x_2, y_2) .

- (ii) When two lines are parallel their gradients are equal. ($m_1 = m_2$)

So the gradient of the line parallel to PQ is also $-\frac{3}{4}$.

- (iii) When two lines are perpendicular $m_1 m_2 = -1$.

$$\text{So } -\frac{3}{4} m_2 = -1$$

$$\Rightarrow m_2 = \frac{4}{3}$$



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The gradient of a line perpendicular to PQ is $\frac{4}{3}$.



Example 4

A straight line L has equation $y = 2x - 5$.

- Find the equation of the line parallel to L and passing through (3, -1).
- Find the equation of the line perpendicular to L and passing through (3, -1).

Solution

Line L has gradient 2.

- Any line parallel to L has gradient 2.

The equation of the line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-1) = 2(x - 3)$$

$$\Rightarrow y + 1 = 2x - 6$$

$$\Rightarrow y = 2x - 7$$

$m = 2$ and
 (x_1, y_1) is (3, -1)

You should check that the point (3, -1) satisfies your line. If it doesn't, you must have made a mistake!

- For two perpendicular lines $m_1 m_2 = -1$, so the gradient of the new line is $-\frac{1}{2}$.

The equation of the line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-1) = -\frac{1}{2}(x - 3)$$

$$\Rightarrow -2y - 2 = x - 3$$

$$\Rightarrow -2y = x - 1$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2}$$

$m = -\frac{1}{2}$ and
 (x_1, y_1) is (3, -1)

The final form of the equation can be written in various different ways:

e.g. $2y = -x + 1$ (This form has no fractions.)

e.g. $2y + x = 1$ (This has no fractions and avoids having a negative sign at the start of the right hand side.)

The **perpendicular bisector** of a line joining two points A and B is the line that is perpendicular to AB and passes through the midpoint of AB (i.e. it **bisects** AB).



Example 5

Find the equation of the perpendicular bisector of the points A (3, 2) and B (-1, 8).

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Solution

$$\text{Gradient of AB} = \frac{8-2}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$$

$$\text{Gradient of perpendicular line} = \frac{2}{3}$$

$$\text{Midpoint of AB is } \left(\frac{3+(-1)}{2}, \frac{2+8}{2} \right) = (1, 5)$$

The equation of the line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 5 = \frac{2}{3}(x - 1)$$

$$\Rightarrow 3y - 15 = 2x - 2$$

$$\Rightarrow 3y = 2x + 13$$