# AQA Level 2 Further Mathematics Calculus

## Section 1: Introduction to differentiation

### Solutions to Exercise

1. (i) $y = 2x + 1$

$$\frac{dy}{dx} = 2$$

(ii) $y = x^3 - 5x$

$$\frac{dy}{dx} = 3x^2 - 5$$

(iii) $y = x(x+2) = x^2 + 2x$

$$\frac{dy}{dx} = 2x + 2$$

2. (i) $y = 3x^2 - 4x + 1$

$$\frac{dy}{dx} = 6x - 4$$

(ii) $y = (x+2)(x-1)$

$= x^2 + x - 1$

$$\frac{dy}{dx} = 2x + 1$$

(iii) $y = x^6(x-1)$

$= x^7 - x^6$

$$\frac{dy}{dx} = 7x^6 - 6x^5$$

3. $y = 2x^5 - 3x^3 - x^2 + 3x$

$$\frac{dy}{dx} = 10x^4 - 9x^2 - 2x + 3$$

When $x = -1$, $\frac{dy}{dx} = 10(-1)^4 - 9(-1)^2 - 2(-1) + 3$

$= 10 - 9 + 2 + 3$

$= 6$

The rate of change of $y$ with respect to $x$ when $x = -1$ is 6.

# AQA FM Calculus 1 Exercise solutions

4. $y = (2x-3)(x^2+1)$

$= 2x^3 - 3x^2 + 2x - 3$

$\frac{dy}{dx} = 6x^2 - 6x + 2$

When $x = 2$, $\frac{dy}{dx} = 6 \times 2^2 - 6 \times 2 + 2$

$= 24 - 12 + 2$

$= 14$

The rate of change of $y$ with respect to $x$ when $x = 2$ is 14.

5. (i) $y = 12x - x^3$

$\frac{dy}{dx} = 12 - 3x^2$

When $x = 0$, $\frac{dy}{dx} = 12$

The gradient of the curve at the origin is 12.

(ii) When gradient is zero, $12 - 3x^2 = 0$

$4 - x^2 = 0$

$(2+x)(2-x) = 0$

$x = -2$ or $x = 2$

When $x = -2$, $y = 12 \times -2 - (-2)^3 = -24 + 8 = -16$

When $x = 2$, $y = 12 \times 2 - 2^3 = 24 - 8 = 16$

The gradient is zero at (-2, -16) and (2, 16).

6. $y = x^4 - x + 1$

$\frac{dy}{dx} = 4x^3 - 1$

When $x = 1$, $\frac{dy}{dx} = 4 \times 1^3 - 1 = 4 - 1 = 3$

When $x = 1$, $y = 1^4 - 1 + 1 = 1$

The tangent is the straight line with gradient 3 passing through (1, 1).

Equation of tangent is $y - 1 = 3(x - 1)$

$y - 1 = 3x - 3$

$y = 3x - 2$

7. $y = x^2 - x$

$$\frac{dy}{dx} = 2x - 1$$

When $x = 3$, $\frac{dy}{dx} = 2\times 3 - 1 = 5$

Gradient of tangent $= 5$, so gradient of normal $= -\frac{1}{5}$.

The normal is the straight line with gradient $-\frac{1}{5}$ passing through (3, 6).

Equation of normal is
$$\begin{aligned} y - 6 &= -\tfrac{1}{5}(x-3) \\ 5(y-6) &= -(x-3) \\ 5y - 30 &= -x + 3 \\ 5y + x &= 33 \end{aligned}$$

Where the normal meets the x-axis, $y = 0$ so $x = 33$.

The normal meets the x-axis at (33, 0).

8. $y = x^3 + 2x^2$

$$\frac{dy}{dx} = 3x^2 + 4x$$

When gradient is 4,
$$\begin{aligned} 3x^2 + 4x &= 4 \\ 3x^2 + 4x - 4 &= 0 \\ (3x-2)(x+2) &= 0 \\ x = \tfrac{2}{3} \text{ or } x &= -2 \end{aligned}$$

9. (i) When $x = 1$, $y = 2x^3 = 2\times 1^3 = 2$

When $x = 1$, $y = 3x^2 - 1 = 3\times 1^2 - 1 = 2$

so the point (1, 2) lies on both curves.

(ii) $y = 2x^3 \Rightarrow \frac{dy}{dx} = 6x$

When $x = 1$, gradient $= 6\times 1 = 6$

$y = 3x^2 - 1 \Rightarrow \frac{dy}{dx} = 6x$

When $x = 1$, gradient $= 6\times 1 = 6$

so the curves have the same gradient at this point.

(iii) The two curves touch each other at (1, 2).

10. $y = ax^3 + bx$

When $x = 1$, $y = a + b \quad \Rightarrow a + b = 8$

$$\frac{dy}{dx} = 3ax^2 + b$$

When $x = 1$, gradient $= 3a + b \quad \Rightarrow 3a + b = 12$

$$3a + b = 12$$
$$a + b = 8$$

Subtracting: $\quad 2a = 4$

$$a = 2, b = 6$$

11. $y = x^3 + x + 2$

$$\frac{dy}{dx} = 3x^2 + 1$$

When $x = 1$, $\frac{dy}{dx} = 3 \times 1^2 + 1 = 4$

When $x = 1$, $y = 1^3 + 1 + 2 = 4$

The tangent has gradient 4 and passes through the point (1, 4).

Equation of tangent is $y - 4 = 4(x - 1)$

$$y - 4 = 4x - 4$$
$$y = 4x$$

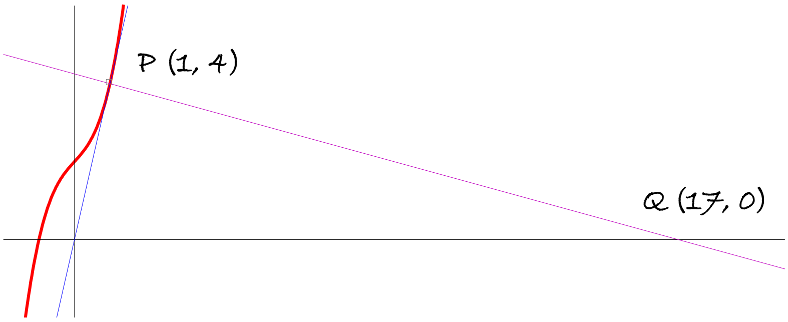
So the tangent passes through the origin.

Gradient of normal $= -\frac{1}{4}$

Equation of normal is $y - 4 = -\frac{1}{4}(x - 1)$

$$4(y - 4) = -(x - 1)$$
$$4y - 16 = -x + 1$$
$$4y + x = 17$$

When $y = 0$, $x = 17$, so Q is (17, 0).




Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 17 \times 4 = 34$

12. $\frac{dy}{dx} = 2x$

The gradient of the tangent at the point (3, 9) is 2 x 3 = 6.

The equation of the tangent is $(y - 9) = 6(x - 3)$

$$y = 6x - 9$$

This crosses the y-axis at the point (0, 9)

13. (i) The normal is perpendicular to the tangent so angles BOD and BCD are right angles.

A circle with diameter BD would pass through O and C (angle in a semicircle is 90°).

(ii) $\frac{dy}{dx} = 2x - 4$

At (0, 0) the gradient of the tangent is -4.

The equation of the tangent is y = -4x.

At (4, 0) the gradient of the tangent is 2×4 – 4 = 4

The equation of the tangent is $(y - 0) = 4(x - 4)$

$y = 4x - 16$

$$y = -4x$$
$$y = 4x - 16$$
$$2y = -16$$
$$y = -8$$
$$-8 = -4x$$
$$x = 2$$

D is the point (2, -8)

At (0, 0) the gradient of the normal is ¼.

The equation of the normal is y = ¼x.

At (4, 0) the gradient of the normal is -¼

The equation of the normal is $(y - 0) = -\frac{1}{4}(x - 4)$

$y = -\frac{1}{4}x + 1$

$$y = \tfrac{1}{4}x$$
$$y = -\tfrac{1}{4}x + 1$$
$$2y = 1$$
$$y = 0.5$$

$0.5 = \frac{1}{4}x$

$x = 2$

B is the point (2, 0.5).

The centre of the circle is the midpoint of BD (2, -3.75).
BD is the diameter of the circle; it has length 8.5. The radius of the circle is 4.25.