Section 2: Further differentiation

Exercise

1. A curve has equation $y = x^3 + 6x^2 + 9x$.
   (i) Differentiate the function to obtain $\frac{dy}{dx}$.
   (ii) Find the x co-ordinates of the points where $\frac{dy}{dx} = 0$ and hence the co-ordinates of the stationary points on the curve.
   (iii) By considering the sign of $\frac{dy}{dx}$ on either side of the stationary points, determine whether the stationary points are maximum or minimum points.
   (iv) Sketch the curve showing the stationary points and points of intersection with the axes clearly.

2. The equation of a curve is given by $y = 2x + x^2 - 4x^3$.
   (i) Find the co-ordinates of the stationary points on the curve, and distinguish between them by considering the gradient on either side of the stationary points.
   (ii) Sketch the curve marking the stationary points and points of intersection with the axes clearly.

3. A curve has equation $y = x^3 - 3x^2 + 6$. Find the co-ordinates of any stationary points and determine their nature showing clearly how your decisions were made. Sketch the curve.

4. The equation of a curve is $y = (x + 1)(x - 3)^3$.
   (i) Write the equation of the curve in the form $y = ax^4 + bx^3 + cx^2 + dx + e$.
   (ii) Find the co-ordinates of the points where $\frac{dy}{dx} = 0$.
   (iii) Classify the stationary points.
   (iv) Sketch the curve.

5. Find the stationary points on the curve $y = x^4 - 2x^3$ and distinguish between them, showing all of the relevant working clearly. Sketch the curve.

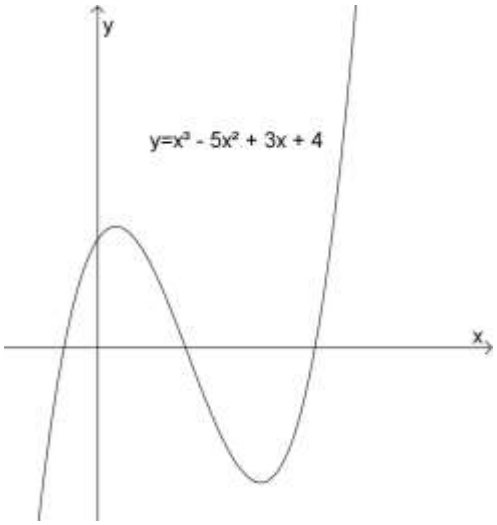
6. The curve $y = x^3 + px^2 + q$ has a minimum point at (4, -11). Find the co-ordinates of the maximum point on the curve.

7. The curve $y = x^3 + ax^2 + bx + c$ passes through the point (1, 1).
   (i) Construct an equation connecting a, b and c.

   The curve also has stationary points when $x = -1$ and when $x = 3$.
   (ii) Construct two further equations connecting a, b and c.
   (iii) Solve the three equations simultaneously to obtain values for a, b and c.

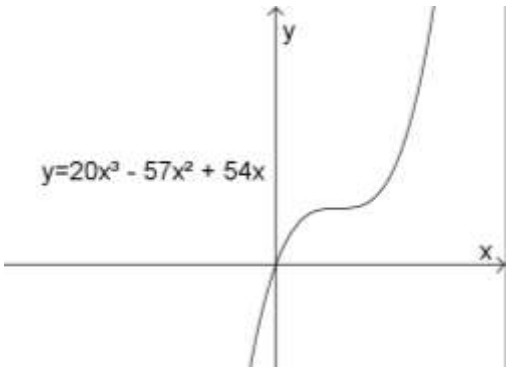
AQA FM Calculus 2 Exercise

8. A sketch of the curve $y = x^3 - 5x^2 + 3x + 4$ is shown below.



Find the set of values of x for which the value of y is less than that at the minimum stationary point.

9. A sketch of the curve $y = 20x^3 - 57x^2 + 54x$ is shown below.



Decide whether the curve has one or two stationary points. You must show all your reasoning.

10. Work out $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following functions:

(i) $y = 10 - 3x - x^2$
(ii) $y = 3x(x^2 - 2x)$
(iii) $y = (2x + 5)(x^2 - 3x)$

11. For the curve $y = -2x^3 + 6x^2 + x - 7$

(i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
(ii) Find the gradient of the curve at the points (1, -2) , (3, -4) and (-1, 0)
(iii) Work out the rate of change of the gradient at each of these points.

