Section 1: Solving linear and quadratic equations

Solutions to Exercise

1. (i) $2x-3=8$

$2x=11$

$x=5.5$

(ii) $3y+2=y-5$

$2y+2=-5$

$2y=-7$

$y=-3.5$

(iii) $3-2a=3a-1$

$3=5a-1$

$4=5a$

$a=0.8$

(iv) $3(p-3)=2(2p+1)$

$3p-9=4p+2$

$-9=p+2$

$-11=p$

$p=-11$

(v) $2(1-z)+3(z+3)=4z+1$

$2-2z+3z+9=4z+1$

$11+z=4z+1$

$11=3z+1$

$10=3z$

$z=\frac{10}{3}$

(vi) $\frac{2b+1}{5}=\frac{3-b}{4}$

$4(2b+1)=5(3-b)$

$8b+4=15-5b$

$13b+4=15$

$13b=11$

$b=\frac{11}{13}$

2. Let the smallest angle be $x°$.
The largest angle is $3x°$.
The third angle is $(x + 20)°$.
The three angles add up to $180°$.

$$x + 3x + (x + 20) = 180$$
$$5x + 20 = 180$$
$$5x = 160$$
$$x = 32$$

The angles are $32°$, $96°$ and $52°$.

(Check: $32 + 96 + 52 = 180$).

3. Let the number of tables which seat 4 people be x.
The number of tables which seat 6 people is $24 - x$.
Total number of seats $= 4x + 6(24 - x)$

$$4x + 6(24 - x) = 114$$
$$4x + 144 - 6x = 114$$
$$30 = 2x$$
$$x = 15$$

There are 15 tables which seat 4 people.

(Check: $15 \times 4 + 9 \times 6 = 60 + 54 = 114$)

4. Let x be the number of boys in the class
So number of girls is $30 - x$.

Total of boys' heights $= 165x$
Total of girls' heights $= 159(30 - x)$
Total of heights for whole class $= 162.2 \times 30 = 4866$

$$165x + 159(30 - x) = 4866$$
$$165x + 4770 - 159x = 4866$$
$$6x = 96$$
$$x = 16$$

There are 16 boys and 14 girls in the class.

5. (i) $x^2 + 4x + 3 = 0$

$$(x + 3)(x + 1) = 0$$
$$x = -3 \text{ or } x = -1$$

(ii) $x^2+5x-6=0$

$(x+6)(x-1)=0$

$x=-6$ or $x=1$

(iii) $x^2-6x+8=0$

$(x-2)(x-4)=0$

$x=2$ or $x=4$

(iv) $x^2-7x-18=0$

$(x-9)(x+2)=0$

$x=9$ or $x=-2$

(v) $2x^2+5x+3=0$

$(2x+3)(x+1)=0$

$x=-\frac{3}{2}$ or $x=-1$

(vi) $2x^2+x-6=0$

$(2x-3)(x+2)=0$

$x=\frac{3}{2}$ or $x=-2$

(vii) $4x^2-3x-10=0$

$(4x+5)(x-2)=0$

$x=-\frac{5}{4}$ or $x=2$

(viii) $6x^2-19x+10=0$

$(3x-2)(2x-5)=0$

$x=\frac{2}{3}$ or $x=\frac{5}{2}$

6. (i) $x^2+2x-2=0$

$a=1, b=2, c=-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{12}}{2}=\frac{-2\pm2\sqrt{3}}{2}=-1\pm\sqrt{3}$$

(ii) $x^2-3x+5=0$

$a=1, b=-3, c=5$

$b^2-4ac=(-3)^2-4\times1\times5=9-20=-11$

Negative so there are no real roots.

(iii) $2x^2 + x - 4 = 0$

$a = 2, b = 1, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{33}}{2 \times 2} = \frac{-1 \pm \sqrt{33}}{4}$$

(iv) $2x^2 - 5x - 12 = 0$

$(2x + 3)(x - 4) = 0$

$x = -\frac{3}{2}$ or $x = 4$

(v) $x^2 - 5x - 3 = 0$

$a = 1, b = -5, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{37}}{2}$$

(vi) $3x^2 + x + 1 = 0$

$a = 3, b = 1, c = 1$

$b^2 - 4ac = 1^2 - 4 \times 3 \times 1 = 1 - 12 = -11$

Negative so there are no real roots.

(vii) $4x^2 + 12x + 9 = 0$

$(2x + 3)^2 = 0$

$x = -\frac{3}{2}$

(viii) $4x^2 + 10x + 5 = 0$

$a = 4, b = 10, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{20}}{2 \times 4} = \frac{-10 \pm 2\sqrt{5}}{8} = \frac{-5 \pm \sqrt{5}}{4}$$

7. Let x be the width of the rectangle, so the length is $x + 3$.

Area $= x(x + 3)$

$x(x + 3) = 40$

$x^2 + 3x = 40$

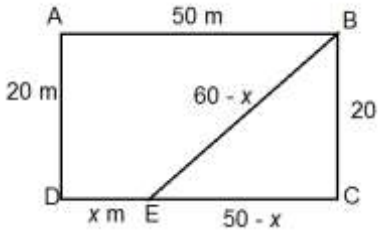
$x^2 + 3x - 40 = 0$

$(x + 8)(x - 5) = 0$

$x = -8$ or 5

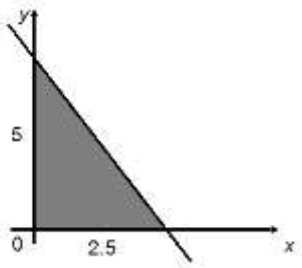
Dimensions must be positive, so width is 5 cm and length is 8 cm.

8.



$$(60-x)^2 = (50-x)^2 + 20^2$$
$$3600 - 120x + x^2 = 2500 - 100x + x^2 + 400$$
$$700 = 20x$$
$$x = 35$$

9. (a) If $a = 0$, the vertical line is on the x-axis and there is no area so $a > 0$.
If $a = 2.5$, the whole triangle below is shaded.



The area of the triangle is 6.25, which is too big so $a < 2.5$.
So $0 < a < 2.5$

(b) The shaded area is $\frac{1}{2}(5+5-2a)a = \frac{a}{2}(10-2a) = 5a - a^2$

$$5a - a^2 = 3$$
$$a^2 - 5a + 3 = 0$$
$$a = \frac{5 \pm \sqrt{25-12}}{2}$$
$$a = \frac{5 \pm \sqrt{13}}{2}$$

$0 < a < 2.5$ so $\frac{5-\sqrt{13}}{2}$