

## Section 1: Basic algebra and simple linear equations

### Notes and examples

These notes contain subsections on

- [Number](#)
- [Manipulating algebraic expressions](#)
- [Collecting like terms](#)
- [Expanding brackets](#)
- [Solving linear equations](#)

### Number

You need to be confident in working with numbers, including fractions, decimals, ratio and percentages. The notes and examples below remind you of some of the skills you need.

#### Fractions

- When adding or subtracting fractions, you need to use a common denominator. For mixed fractions you can deal with the whole numbers separately – but make sure you tidy up at the end if the fraction part is top-heavy.
- To multiply fractions, mixed numbers must be changed into top-heavy fractions before multiplying the numerators and the denominators. It's easier to do any 'cancelling' before doing the multiplication.
- When dividing, you must also change mixed numbers into top-heavy fractions. Dividing by a fraction is the same as multiplying by its reciprocal.



#### Example 1

Work out:

(i)  $2\frac{3}{4} + 1\frac{2}{3}$

(ii)  $5\frac{1}{6} - 2\frac{5}{8}$

(iii)  $2\frac{1}{4} \times 1\frac{2}{3}$

(iv)  $1\frac{1}{3} \div 1\frac{3}{5}$

**Solution**

$$\begin{aligned}
 \text{(i)} \quad 2\frac{3}{4} + 1\frac{2}{3} &= 2 + \frac{9}{12} + 1 + \frac{8}{12} \\
 &= 3\frac{17}{12} \\
 &= 4\frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad 5\frac{1}{6} - 2\frac{5}{8} &= 5 + \frac{4}{24} - 2 - \frac{15}{24} \\
 &= 3 - \frac{11}{24} \\
 &= 2 + \frac{24}{24} - \frac{11}{24} \\
 &= 2\frac{13}{24}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad 2\frac{1}{4} \times 1\frac{2}{3} &= \frac{9}{4} \times \frac{5}{3} \\
 &= \frac{\cancel{9}^3}{4} \times \frac{5}{\cancel{3}^1} \\
 &= \frac{15}{4} \\
 &= 3\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad 1\frac{1}{3} \div 1\frac{3}{5} &= \frac{4}{3} \div \frac{8}{5} \\
 &= \frac{\cancel{4}^1}{3} \times \frac{5}{\cancel{8}^2} \\
 &= \frac{5}{6}
 \end{aligned}$$

**Percentages**

- To work with percentages, you will usually need to write them as decimals.
- For example, to find 58% of something, you need to multiply by 0.58
- To increase something by, say, 8%, multiply by 1.08 (the 1 gives you the original amount, and the 0.08 gives the extra 8%)
- To decrease something by, say, 24%, multiply by 0.76 (the new amount is 76% of the original amount).



## Example 2

- (i) Find 6% of 240.
- (ii) Amy scores 68 marks out of 80 in a test. What is this as a percentage?
- (iii) A restaurant bill is £32.40. I have a voucher which gives a 15% discount. How much do I pay?
- (iv) Ahmed is paid £7.60 per hour. His pay is increased to £8.20 per hour. What is the percentage increase in his pay (to the nearest whole number)?

## Solution

- (i) 6% as a decimal is 0.06.  
 $240 \times 0.06 = 14.4$
- (ii) To change a fraction into a percentage, multiply by 100.  
 $\frac{68}{80} \times 100 = 85$
- (iii) 15% is taken off the bill, so the final amount is 85% of the original bill.  
 $32.4 \times 0.85 = 27.54$   
 The bill is £27.54
- (iv) Ahmed's pay has been multiplied by  $\frac{8.20}{7.60} = 1.078\dots$   
 As a percentage this is 108% to the nearest whole number.  
 Ahmed's pay is 108% of the previous amount, so it has been increased by 8%.

## Ratio

- Ratios are similar to fractions, in that you can 'cancel' them down to simplify them.



## Example 3

Given that  $x : y = 2 : 3$  and  $y : z = 2 : 5$ , find:

- (i)  $x : z$
- (ii)  $2y : 3z$
- (iii)  $x + y : z$

## Solution

- (i)  $x : y = 4 : 6$   
 $y : z = 6 : 15$   
 so  $x : z = 4 : 15$
- (ii)  $2y : 3z = 4 : 9$
- (iii)  $x + y : y = 5 : 3 = 10 : 6$   
 $y : z = 6 : 15$   
 $x + y : z = 10 : 15 = 2 : 3$



## Manipulating algebraic expressions

Throughout your mathematics you will need to be able to manipulate algebraic expressions confidently. The examples below remind you of the important techniques of collecting like terms, removing brackets, factorising, multiplying, and adding, subtracting and simplifying algebraic fractions.

### Collecting like terms



#### Example 4

Simplify the expression

$$3a + 2b - a + 3b - 2ab + 2a$$

#### Solution

There are three different types of “like term” in this expression. There are terms in  $a$ , terms in  $b$ , and a term in  $ab$ . Notice that the term in  $ab$  cannot be combined with either the terms in  $a$  or the terms in  $b$ , but remains as a term on its own.

$$\begin{aligned} 3a + 2b - a + 3b - 2ab + 2a &= 3a - a + 2a + 2b + 3b - 2ab \\ &= 4a + 5b - 2ab \end{aligned}$$

In the example the expression has been rewritten with each set of like terms grouped together, before simplifying by adding / subtracting the like terms. You may well not need to write down this intermediate stage.

### Expanding brackets

When multiplying out brackets, each term in the bracket must be multiplied by the number or expression outside the bracket.



#### Example 5

Simplify the expressions

(i)  $3(p - 2q) + 2(3p + q)$

(ii)  $2x(x + 3y) - y(2x - 5y)$

#### Solution

(i)  $3(p - 2q) + 2(3p + q) = 3p - 6q + 6p + 2q$   
 $= 9p - 4q$

(ii)  $2x(x + 3y) - y(2x - 5y) = 2x^2 + 6xy - 2xy + 5y^2$   
 $= 2x^2 + 4xy + 5y^2$

Multiplying out two brackets of the form  $(ax + b)(cx + d)$  gives a quadratic function. Make sure that you are confident in this.



## Example 6

Multiply out  $(x+2)(3x-4)$

### Solution

$$\begin{aligned}(x+2)(3x-4) &= 3x^2 - 4x + 6x - 8 \\ &= 3x^2 + 2x - 8\end{aligned}$$

You might also need to multiply out more complicated examples, such as multiplying a quadratic expression by a linear expression. The important point to remember is that each term in the first bracket must be multiplied by each term in the second bracket. So (before simplifying) you can check that you have the right number of terms by multiplying the number of terms in one bracket by the number of terms in the other bracket.

## Example 7

Multiply out  $(2x-3)(x^3-2x^2+3x+4)$

### Solution

$$\begin{aligned}(2x-3)(x^3-2x^2+3x+4) &= 2x(x^3-2x^2+3x+4) - 3(x^3-2x^2+3x+4) \\ &= 2x^4 - 4x^3 + 6x^2 + 8x - 3x^3 + 6x^2 - 9x - 12 \\ &= 2x^4 - 7x^3 + 12x^2 - x - 12\end{aligned}$$

There are 2 terms in the first bracket and 4 in the second, so there will be 8 terms in the expansion

You also need to be able to multiply out expressions involving more than one bracket. This is shown in the next example.

## Example 8

Multiply out  $(x-2)(2x+3)(3x-1)$

### Solution

It is often easiest to multiply out one pair of brackets, and then multiply the result by the third bracket.

$$\begin{aligned}(x-2)(2x+3) &= 2x^2 + 3x - 4x - 6 \\ &= 2x^2 - x - 6\end{aligned}$$

Simplify here or the next step will be more complicated!

$$\begin{aligned}(2x^2 - x - 6)(3x - 1) &= 2x^2(3x - 1) - x(3x - 1) - 6(3x - 1) \\ &= 6x^3 - 2x^2 - 3x^2 + x - 18x + 6 \\ &= 6x^3 - 5x^2 - 17x + 6\end{aligned}$$

## Linear equations

A linear equation involves only terms in  $x$  (or whatever variable is being used) and numbers. So it has no terms involving  $x^2$ ,  $x^3$  etc. Equations like these are called linear because the graph of an expression involving only terms in  $x$  and numbers (e.g.  $y = 2x + 1$ ) is always a straight line.

Solving a linear equation may involve simple algebraic techniques such as gathering like terms and multiplying out brackets. Example 9 shows a variety of techniques that you might need to use.



### Example 9

Solve these equations.

(i)  $5x - 2 = 3x + 8$

(ii)  $3(2y - 1) = 4 - 2(y - 3)$

(iii)  $\frac{2a - 1}{3} = 2a + 3$

### Solution

(i)  $5x - 2 = 3x + 8$

$5x = 3x + 8 + 2$

$5x = 3x + 10$

$5x - 3x = 10$

$2x = 10$

$x = 5$

Add 2 to each side

Subtract  $3x$  from each side

Divide each side by 2

(ii)  $3(2y - 1) = 4 - 2(y - 3)$

$6y - 3 = 4 - 2y + 6$

$6y - 3 = 10 - 2y$

$6y = 13 - 2y$

$8y = 13$

$y = \frac{13}{8}$

Multiply out the brackets

Add 3 to each side

Add  $2y$  to each side

Divide each side by 8

(iii)  $\frac{2a - 1}{3} = 2a + 3$

$2a - 1 = 3(2a + 3)$

$2a - 1 = 6a + 9$

$2a = 6a + 10$

$-4a = 10$

$a = -2.5$

Multiply both sides by 3

Multiply out the brackets

Add 1 to each side

Subtract  $6a$  from each side

Divide both sides by  $-4$



In Example 2, the problem is given in words and you need to express this algebraically before solving the equation.

**Example 2**

Jamila has a choice of two tariffs for text messages on her mobile phone.

Tariff A: 10p for the first 5 messages each day, 2p for all others

Tariff B: 4p per message

How many messages would Jamila need to send each day for the two tariffs to cost the same?  
(She always sends at least 5!)

**Solution**

Let the number of messages Jamila sends per day be  $n$ .

Under Tariff A, she has to pay 10p for each of 5 messages and 2p for each of  $n - 5$  messages.

$$\text{Cost} = 50 + 2(n - 5)$$

Under Tariff B, she has to pay 4p for each of  $n$  messages.

$$\text{Cost} = 4n$$

For the cost to be the same

$$50 + 2(n - 5) = 4n$$

$$50 + 2n - 10 = 4n$$

$$40 + 2n = 4n$$

$$40 = 2n$$

$$20 = n$$

She needs to send 20 messages per day for the two tariffs to cost the same.

