AQA Level 2 Further Mathematics Algebra IV

Section 2: Simultaneous equations

Notes and Examples

These notes contain subsections on
- **Linear simultaneous equations using elimination**
- **Linear simultaneous equations using substitution**
- **One linear and one quadratic equation**
- **Three simultaneous equations**

Linear simultaneous equations using elimination

This work may be revision. You need to make sure that you can solve linear simultaneous equations confidently before you move on to the work on one linear and one quadratic equation, which will probably be new to you.

Simultaneous equations involve more than one equation and more than one unknown. To solve them you need the same number of equations as there are unknowns.

One method of solving simultaneous equations involves adding or subtracting multiples of the two equations so that one unknown disappears. This method is called *elimination*, and is shown in the next example.

Example 1
Solve the simultaneous equations
$$3p + q = 5$$
$$p - 2q = 4$$

Solution

① $\qquad 3p + q = 5$

② $\qquad p - 2q = 4$

> Adding or subtracting these equations will not eliminate either p or q. However, you can multiply the first equation by 2, and then add. This will eliminate p.

① × 2 $\qquad 6p + 2q = 10$

② $\qquad\quad p - 2q = 4$

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Adding: $\qquad 7p = 14$

$\qquad\qquad\quad p = 2$

$\qquad 3 \times 2 + q = 5$

$\qquad\quad 6 + q = 5$

$\qquad\qquad\quad q = -1$

> Now substitute this value for p into one of the original equations – you can use either, but in this example, ①

The solution is $p = 2$, $q = -1$.
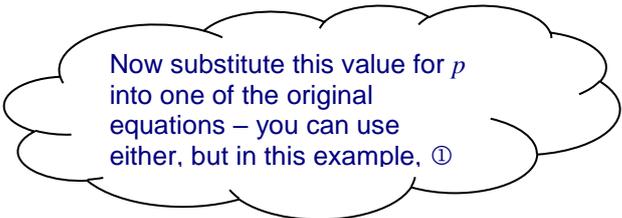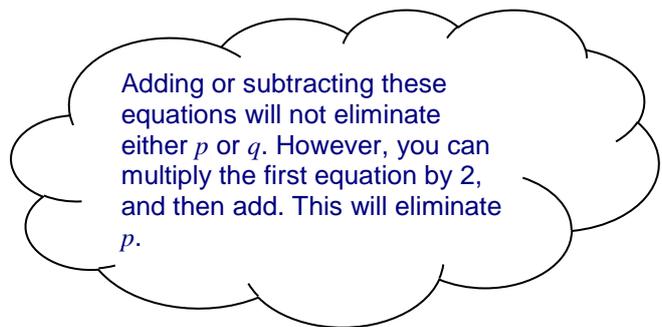
Notice that, in Example 1, you could have multiplied equation ② by 3 and then subtracted. This would give the same answer.

Sometimes you need to multiply each equation by a different number before you can add or subtract. This is the case in the next example.

Example 2
5 pencils and 2 rubbers cost £1.50
8 pencils and 3 rubbers cost £2.35
Find the cost of a pencil and the cost of a rubber.

Solution

① $\quad 5p + 2r = 150$

② $\quad 8p + 3r = 235$

> Let p represent the cost of a pencil and r represent the cost of a rubber. It is easier to work in pence.

① × 3 $\quad 15p + 6r = 450$

② × 2 $\quad 16p + 6r = 470$

> The easiest method is to multiply equation ① by 3 and equation ② by 2. (You could of course multiply ① by 8 and ② by

Subtracting: $\quad -p = -20$

$\quad p = 20$

$5 \times 20 + 2r = 150$

$100 + 2r = 150$

> Substitute this value of p into equation ①

$2r = 50$

$r = 25$

A pencil costs 20p and a rubber costs 25p.

Linear simultaneous equations using substitution

An alternative method of solving simultaneous equations is called substitution. This can be the easier method to use in cases where one equation gives one of the variables in terms of the other. This is shown in the next example.

Example 3
Solve the simultaneous equations
$\quad 3x - 2y = 11$
$\quad y = 5 - 2x$

Solution

> Substitute the expression for y given in the second equation, into the first equation

$$3x - 2(5 - 2x) = 11$$
$$3x - 10 + 4x = 11$$
$$7x = 21$$
$$x = 3$$
$$y = 5 - 2 \times 3$$
$$= 5 - 6$$
$$= -1$$

Multiply out the brackets

Substitute the value for x into the original second

The solution is $x = 3$, $y = -1$

One linear and one quadratic equation

When you need to solve a pair of simultaneous equations, one of which is linear and one of which is quadratic, you need to substitute the linear equation into the quadratic equation.

Example 4
Solve the simultaneous equations
$$x^2 + 2y^2 = 6$$
$$x - y = 1$$

Solution

Start by using the linear equation to write one variable in terms of the other.

$$x = y + 1$$

Now substitute this expression for y into the first equation

$$(y + 1)^2 + 2y^2 = 6$$
$$y^2 + 2y + 1 + 2y^2 = 6$$
$$3y^2 + 2y - 5 = 0$$
$$(3y + 5)(y - 1) = 0$$
$$y = -\tfrac{5}{3} \text{ or } y = 1$$

Multiply out, simplify and factorise

$y = -\tfrac{5}{3}$	$x = y + 1 = -\tfrac{5}{3} + 1 = -\tfrac{2}{3}$
$y = 1$	$x = y + 1 = 1 + 1 = 2$

Sometimes you will need to use the quadratic formula to solve the resulting quadratic

Now substitute each value for y into the linear equation to find the corresponding values of

The solutions are $x = -\tfrac{2}{3}, y = -\tfrac{5}{3}$ and $x = 2, y = 1$

Three simultaneous equations

In order to solve a set of three simultaneous equations with 3 unknowns you need to combine the equations in pairs so that you end up with 2 equations with the same two variables. Then you can solve these simultaneously as above to find two of the variables. By substituting these into one of the original equations you can find the final variable.

Example 5

Solve the simultaneous equations

① $\quad 5x + y - z = 6$

② $\quad 2x + 3y + 2z = 7$

③ $\quad 3x - 2y - 2z = 2$

Solution

① × 2
$$10x + 2y - 2z = 12$$
②
$$2x + 3y + 2z = 7$$
$$\overline{12x + 5y = 19} \quad ④$$

> Here you might choose to eliminate z by adding two lots of equation 1 to equation 2, you could then add equation 3 to equation 2.

②
$$2x + 3y + 2z = 7$$
③
$$3x - 2y - 2z = 2$$
$$\overline{5x + y = 9} \quad ⑤$$

> You should now have two equations in two unknowns, you can now solve these equations like you would any pair of simultaneous equations.

⑤ × 5
$$25x + 5y = 45$$
④
$$12x + 5y = 19$$
$$\overline{13x = 26}$$
$$x = 2$$

> Now substitute this value for x into ⑤ to find y.

④
$$5(2) + y = 9$$
$$10 + y = 9$$
$$y = -1$$

①
$$5(2) + (-1) - z = 6$$
$$10 - 1 - z = 6$$
$$3 = z$$

> Now you can substitute these values for x and x in any of the original equations to find z.

The solution is $x = 2$, $y = -1$ and $z = 3$.