

Section 3: Exponentials functions

Exercise solutions

1 (i)

x	0	1	2
$y = a^x$	1	2	4

(ii) since $y = a^1 = 2$, $a = 2$

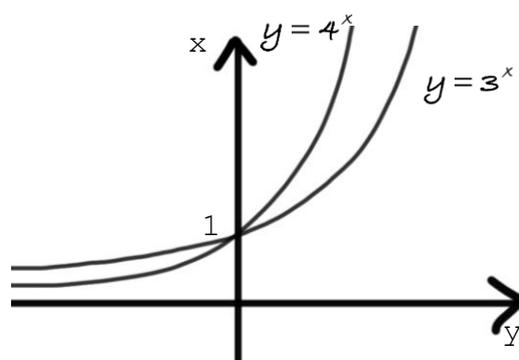
(iii) at $x=4$, $y = 2^4 = 16$

at $x=-2$, $y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

1. (i) Both curves must go through $(0,1)$, which is labelled.

Neither graph touches or crosses the x-axis.

$y = 4^x$ is steeper than $y = 3^x$ and both curves are labelled

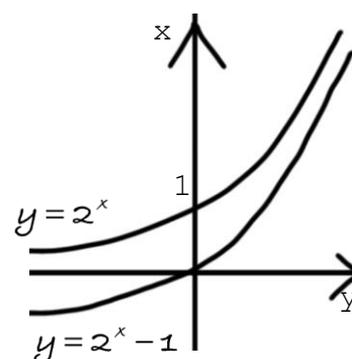


(ii) $y = 2^x$ crosses the y-axis at 1.

$y = 2^x - 1$ passes through the origin.

The graphs should not meet.

Both curves are labelled

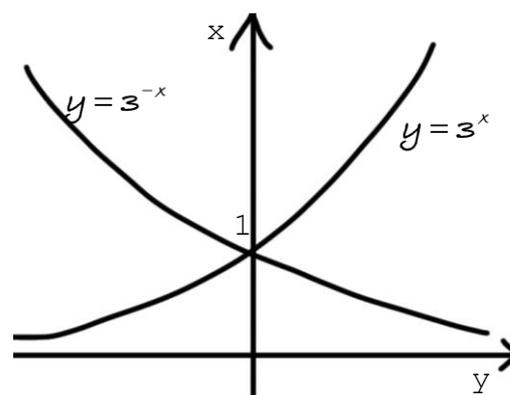


(iii) The graphs should look symmetric across the x axis.

Both pass through $(0,1)$, which is labelled.

Neither graph touches or crosses the x-axis.

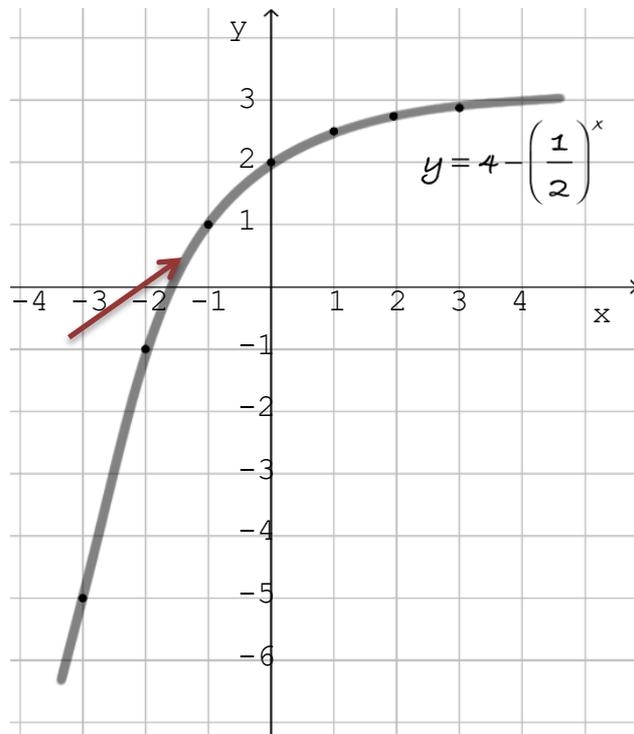
Both curves are labelled.



2. (i)

x	-3	-2	-1	0	1	2	3
$y = 4 - \left(\frac{1}{2}\right)^x$	-5	-1	1	2	2.5	2.75	2.875

(ii)
 Points plotted
 Graph labelled
 Smooth curve drawn



(iii)
 Some indication of having read from the graph.

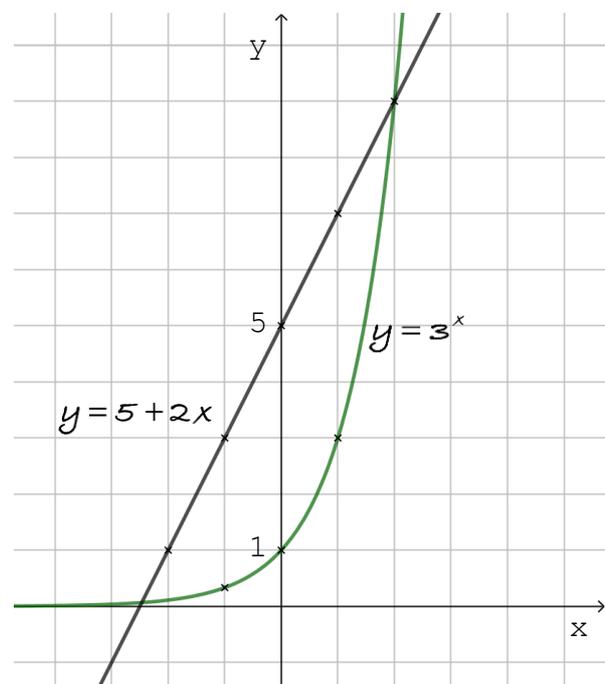
Solution in range
 $-1.8 \leq x \leq -1.3$

(More accuracy than 1dp suggests solving by a different method e.g. $x = -1.58496\dots$)

3.

(i)
 Graph shows:
 -Line passing through (0,5).
 -Curve passes through (0,1).

(ii)
 As the graphs intersect twice, there will be 2 solutions.



4. (i) $y = a \times b^x$

at $x = -1$, $y = 5$, so:

$$5 = a \times b^{-1}$$

$$5 = \frac{a}{b}$$

$$a = 5b$$

also, at $x = 5$, $y = 3645$

$$3645 = a \times b^5$$

$$3645 = 5b \times b^5$$

$$\frac{3645}{5} = b^6$$

$$729 = b^6$$

$$b = \sqrt[6]{729}$$

$$b = 3$$

since $a = 5b$,

$$a = 15$$

(ii) $y = 135$ where $x = c$:

$$y = 15 \times 3^x$$

$$135 = 15 \times 3^c$$

$$\frac{135}{15} = 3^c$$

$$9 = 3^c$$

$$c = 2$$

5. (i) $P(t) = 40000 \times (1.2)^t$

(ii) $P(5) = 40000 \times (1.2)^5 = 99533$

6. (i) $P(324) = 1013 \times 0.88^{(324/1000)} = 972$ millibars

(ii) $P(1345) = 1013 \times 0.88^{(1345/1000)} = 853$ millibars

7. (i) at $x=0$, $N = 100 \times 3^{\left(\frac{0}{24}\right)} = 100 \times 3^0 = 100 \times 1 = 100$

(ii) after 2 days, 48 hours have passed.

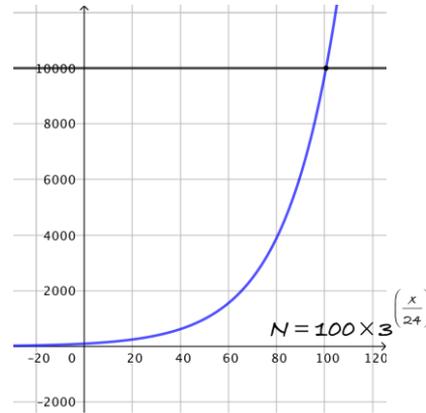
at $x=48$, $N = 100 \times 3^{\left(\frac{48}{24}\right)} = 100 \times 3^2 = 100 \times 9 = 900$

(iii) Plot a graph of $N = 100 \times 3^{\left(\frac{x}{24}\right)}$

Reading off the graph at 10 000 gives an answer of approx. 100.

At $x=100$, $N = 100 \times 3^{\left(\frac{100}{24}\right)} = 9728$

At $x=101$, $N = 100 \times 3^{\left(\frac{101}{24}\right)} = 10183$



We would expect it to take 4 days and 5 hours to reach 10 000 bacteria.