

## Section 3: Trig graphs, identities and equations

### Solutions to Exercise

$$1. \quad (i) \quad \frac{\sqrt{1 - \cos^2 x}}{\tan x} = \frac{\sqrt{\sin^2 x}}{\tan x}$$

$$= \sin x \times \frac{\cos x}{\sin x} = \cos x$$

$$(ii) \quad \frac{\sin x}{\sqrt{1 - \sin^2 x}} = \frac{\sin x}{\sqrt{\cos^2 x}}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

$$(iii) \quad \frac{\cos^2 x}{1 + \sin x} = \frac{1 - \sin^2 x}{1 + \sin x}$$

$$= \frac{(1 + \sin x)(1 - \sin x)}{1 + \sin x}$$

$$= 1 - \sin x$$

2. (i)  $\sin x = 0.3$   
Solutions are in the first and second quadrants  
 $x = 17.5^\circ$  or  $180^\circ - 17.5^\circ$   
 $x = 17.5^\circ$  or  $162.5^\circ$

(ii)  $\tan x = 1.5$   
Solutions are in the first and third quadrants  
 $x = 56.3^\circ$  or  $180^\circ + 56.3^\circ$   
 $x = 56.3^\circ$  or  $236.3^\circ$

(iii)  $\cos x = -0.7$   
Solutions are in the second and third quadrant  
 $x = 180^\circ - 45.6^\circ$  or  $180^\circ + 45.6^\circ$   
 $x = 134.4^\circ$  or  $225.6^\circ$

(iv)  $\sin x = -0.6$   
Solutions are in the third and fourth quadrant  
 $x = 180^\circ + 36.9^\circ$  or  $360^\circ - 36.9^\circ$   
 $x = 216.9^\circ$  or  $323.1^\circ$

## AQA FM Geometry I 3 Exercise solutions

3. (i)  $\sin x = 0.6$   
Solutions are in the first and second quadrants  
 $x = 36.9^\circ$  or  $180^\circ - 36.9^\circ$   
 $x = 36.9^\circ$  or  $143.1^\circ$
- (ii)  $\cos x = 0.8$   
Solutions are in the first and fourth quadrants  
 $x = 36.9^\circ$  or  $360^\circ - 36.9^\circ$   
 $x = 36.9^\circ$  or  $323.1^\circ$
- (iii)  $\tan x = -0.6$   
Solutions are in the second and fourth quadrants  
 $x = 180^\circ - 31.0^\circ$  or  $360^\circ - 31.0^\circ$   
 $x = 149.0^\circ$  or  $329.0^\circ$
- (iv)  $\cos x = -0.3$   
Solutions are in the second and third quadrants  
 $x = 180^\circ - 72.5^\circ$  or  $180^\circ + 72.5^\circ$   
 $x = 107.5^\circ$  or  $252.5^\circ$
4. (i)  $\sin x = \frac{1}{\sqrt{2}}$   
 $\sin 45^\circ = \frac{1}{\sqrt{2}}$ , and solutions are in the first and second quadrants  
so  $x = 45^\circ$  or  $180^\circ - 45^\circ$   
 $x = 45^\circ$  or  $135^\circ$
- (ii)  $\sin x = -\frac{1}{2}$   
 $\sin 30^\circ = \frac{1}{2}$ , and solutions are in the third and fourth quadrants  
so  $x = 180^\circ + 30^\circ$  or  $360^\circ - 30^\circ$   
 $x = 210^\circ$  or  $330^\circ$
- (iii)  $\cos x = \frac{\sqrt{3}}{2}$   
 $\cos 30^\circ = \frac{\sqrt{3}}{2}$ , and solutions are in the first and fourth quadrants  
so  $x = 30^\circ$  or  $360^\circ - 30^\circ$   
 $x = 30^\circ$  or  $330^\circ$

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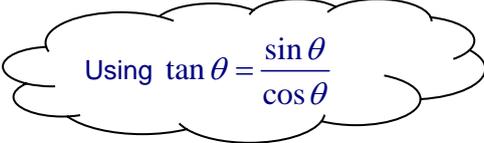
(iv)  $\cos x = -\frac{1}{\sqrt{2}}$   
 $\cos 45^\circ = \frac{1}{\sqrt{2}}$ , and solutions are in the second and third quadrants  
 so  $x = 180^\circ - 45^\circ$  or  $180^\circ + 45^\circ$   
 $x = 135^\circ$  or  $225^\circ$

5. (i)  $3 \sin x = 4 \cos x$

$$\tan x = \frac{4}{3}$$

$$x = 53.1^\circ \text{ or } 180^\circ + 53.1^\circ$$

$$x = 53.1^\circ \text{ or } 233.1^\circ$$



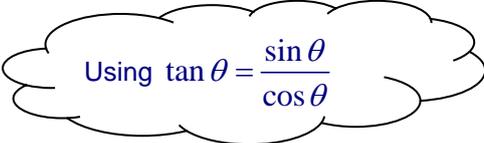
Using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

(ii)  $2 \cos x = -3 \sin x$

$$\tan x = -\frac{2}{3}$$

$$x = 180^\circ - 33.7^\circ \text{ or } 360^\circ - 33.7^\circ$$

$$x = 146.3^\circ \text{ or } 326.3^\circ$$



Using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

6. (i)  $4 \cos^2 \theta = 3$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ \text{ or } 330^\circ$$

$$\cos \theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = 150^\circ \text{ or } 210^\circ$$

$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

(ii)  $2 \cos^2 \theta = \cos \theta$

$$2 \cos^2 \theta - \cos \theta = 0$$

$$\cos \theta (2 \cos \theta - 1) = 0$$

$$\cos \theta = 0 \text{ or } \cos \theta = \frac{1}{2}$$

$$\cos \theta = 0 \Rightarrow \theta = 90^\circ \text{ or } 270^\circ$$

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \text{ or } 300^\circ$$

$$\theta = 60^\circ, 90^\circ, 270^\circ, 300^\circ$$

(iii)  $4 \sin \theta \cos \theta = \sin \theta$

$$4 \sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (4 \cos \theta - 1) = 0$$

$$\sin \theta = 0 \text{ or } \cos \theta = \frac{1}{4}$$

$$\sin \theta = 0 \Rightarrow \theta = 0^\circ \text{ or } 180^\circ \text{ or } 360^\circ$$

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$$\cos \theta = \frac{1}{4} \Rightarrow \theta = 75.5^\circ \text{ or } 284.5^\circ$$

$$\theta = 0^\circ, 75.5^\circ, 180^\circ, 284.5^\circ, 360^\circ$$

$$(iv) \quad \cos^2 \theta - \cos \theta - 2 = 0$$

$$(\cos \theta - 2)(\cos \theta + 1) = 0$$

$$\cos \theta = 2 \text{ or } \cos \theta = -1$$

There are no real values of  $\theta$  for which  $\cos \theta = 2$

$$\cos \theta = -1 \Rightarrow \theta = 180^\circ$$

$$(v) \quad 3 \sin^2 \theta + 5 \cos \theta - 1 = 0$$

$$3(1 - \cos^2 \theta) + 5 \cos \theta - 1 = 0$$

$$3 - 3 \cos^2 \theta + 5 \cos \theta - 1 = 0$$

$$3 \cos^2 \theta - 5 \cos \theta - 2 = 0$$

$$(3 \cos \theta + 1)(\cos \theta - 2) = 0$$

$$\cos \theta = -\frac{1}{3} \text{ or } \cos \theta = 2$$

There are no real values of  $\theta$  for which  $\cos \theta = 2$

$$\cos \theta = -\frac{1}{3} \Rightarrow \theta = 109.5^\circ \text{ or } 250.5^\circ$$

$$(vi) \quad 3 \tan \theta - 2 \cos \theta = 0$$

$$\frac{3 \sin \theta}{\cos \theta} - 2 \cos \theta = 0$$

$$3 \sin \theta - 2 \cos^2 \theta = 0$$

$$3 \sin \theta - 2(1 - \sin^2 \theta) = 0$$

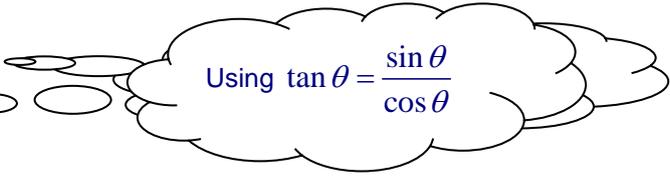
$$2 \sin^2 \theta + 3 \sin \theta - 2 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 2) = 0$$

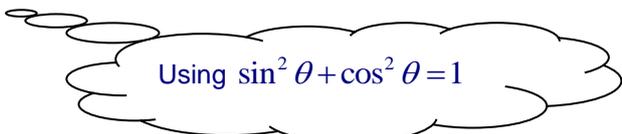
$$\sin \theta = \frac{1}{2} \text{ or } \sin \theta = -2$$

There are no real values of  $\theta$  for which  $\sin \theta = -2$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ \text{ or } 150^\circ$$



$$\text{Using } \tan \theta = \frac{\sin \theta}{\cos \theta}$$



$$\text{Using } \sin^2 \theta + \cos^2 \theta = 1$$

7. Any three angles which are a multiple of  $180^\circ$  either more or less than  $132^\circ$ .

$$8. \quad \frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} = \frac{1 - \sin x + 1 + \sin x}{(1 - \sin x)(1 + \sin x)}$$

$$= \frac{2}{1 - \sin^2 x}$$

$$= \frac{2}{\cos^2 x}$$