Section 2: Graphs of functions

Solutions to Exercise

1. (i) $y = 1 - 3x$ where x can take any value
When $x = 0$, $y = 1$
When $y = 0$, $1 - 3x = 0 \quad \Rightarrow x = \frac{1}{3}$
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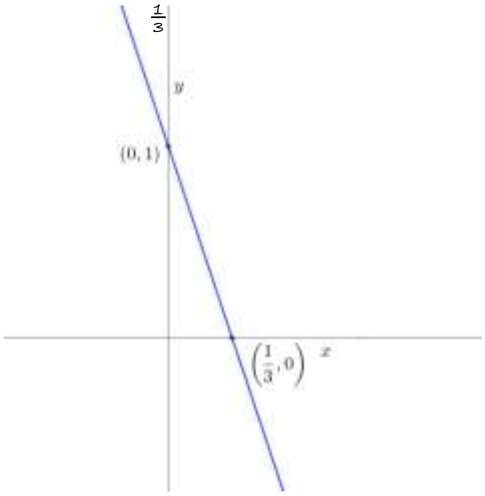
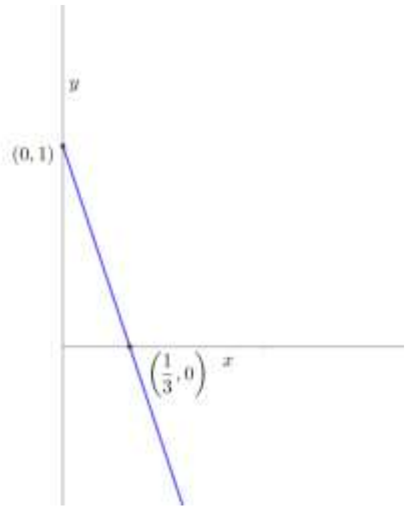


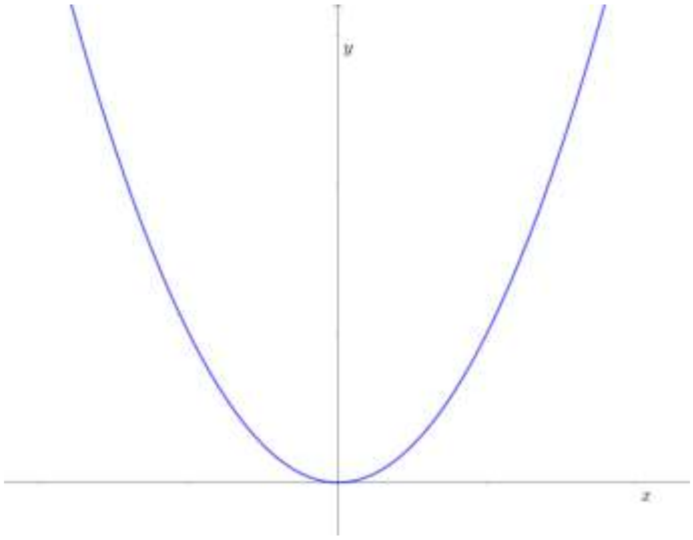
From the graph we can see the range is $f(x) \in \mathbb{R}$.

(ii) $y = 1 - 3x$ where $x > 0$
When $x = 0$, $y = 1$
When $y = 0$, $1 - 3x = 0 \quad \Rightarrow x = \frac{1}{3}$



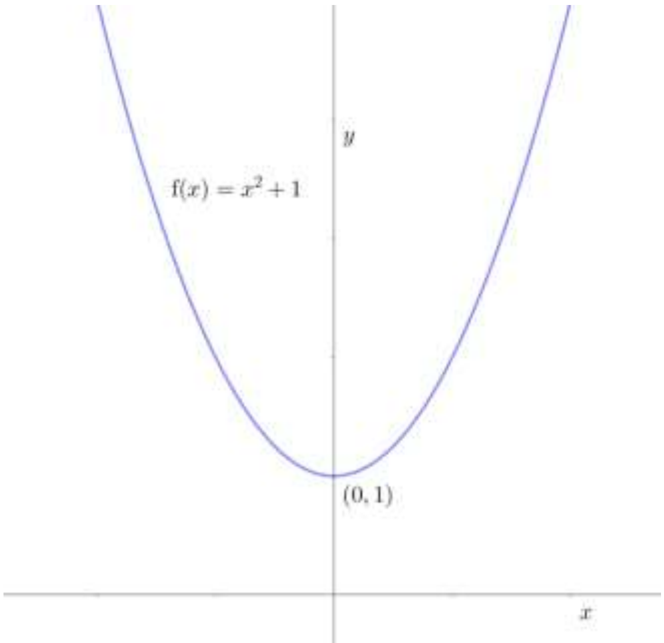
From the graph we can see the range is $f(x) < 1$

(iii) $y = x^2$ where x can take any value
When $x = 0$, $y = 0$
The graph is a positive quadratic with minimum at (0, 0)



The range is $f(x) \geq 0$.

(iv) $f(x) = x^2 + 1$ where x can take any value
When $x = 0$, $y = 1$

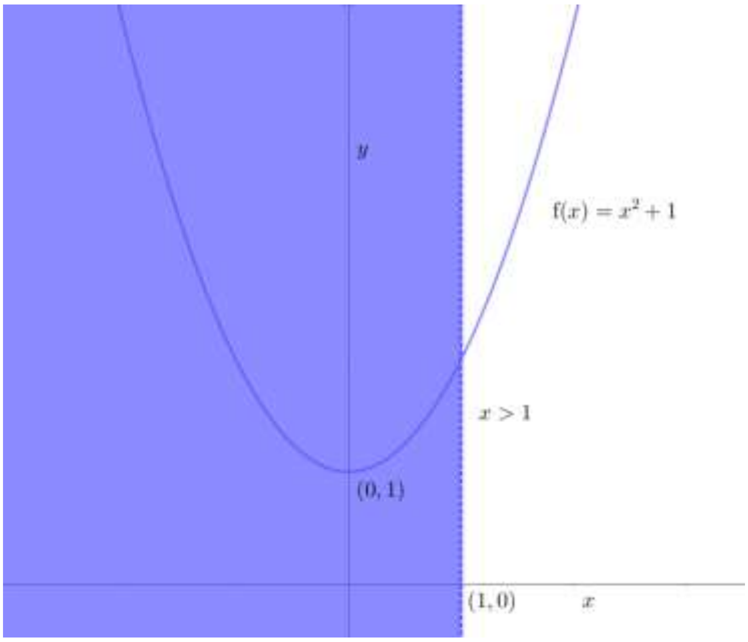


From the graph, we can see the range is $f(x) \geq 1$

(v)

$f(x) = x^2 + 1$ where $x > 1$

Using the same graph as part (iv), we shade out the values for which $x \le 1$



To find the range, look at where the line $x = 1$ meets $f(x) = x^2 + 1$

$f(1) = (1)^2 + 1 = 2$

Since $x > 1$ is a strict inequality, the range of the function is $f(x) > 2$

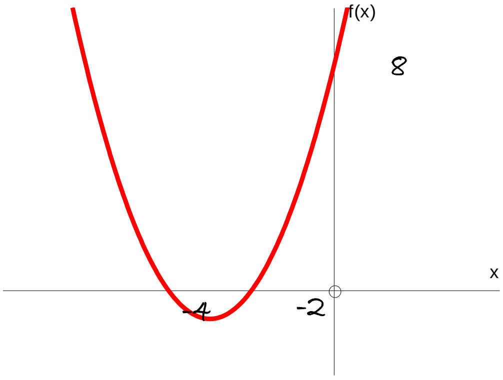
2. (i) $y = x^2 + 6x + 8$

When $x = 0$, $y = 8$

When $y = 0$, $x^2 + 6x + 8 = 0$

$(x+2)(x+4) = 0$

$x = -2$ or -4



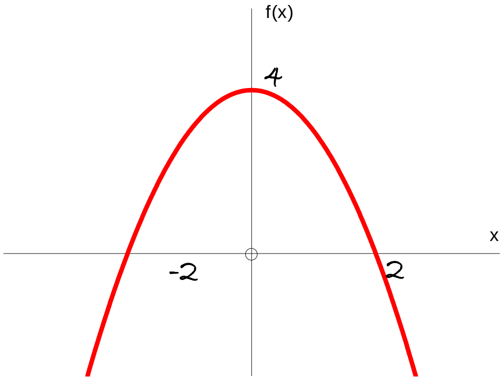
(ii) $y = 4 - x^2$

When $x = 0$, $y = 4$

When $y = 0$, $4 - x^2 = 0$

$$(2 - x)(2 + x) = 0$$

$$x = 2 \text{ or } -2$$



3. (i) $f(-1) = \dfrac{1}{1 + (-1)^2} = \dfrac{1}{2} = 0.5$

$f(\tfrac{1}{2}) = \dfrac{1}{1 + (\tfrac{1}{2})^2} = \dfrac{1}{\tfrac{5}{4}} = \dfrac{4}{5} = 0.8$

(ii) $f(x) = \dfrac{1}{1 + x^2}$ where $-1 \le x \le 1$

The largest possible value of $f(x)$ is when $x = 0$, where $f(x) = 1$.

The smallest possible value of $f(x)$ is when $x = \pm 1$, where $f(x) = \tfrac{1}{2}$.

The range is $\tfrac{1}{2} \le f(x) \le 1$.

4. (i) $x = 1$ must be excluded from the domain, since the function is not defined for this value.

(ii) (a) $f(2) = \dfrac{1}{2 - 1} = 1$

(b) $f(-3) = \dfrac{1}{-3 - 1} = -\dfrac{1}{4}$

(c) $f(0) = \dfrac{1}{0 - 1} = -1$

(iii) $f(x) = 2$

$\dfrac{1}{x-1} = 2$

$1 = 2(x-1)$

$1 = 2x - 2$

$2x = 3$

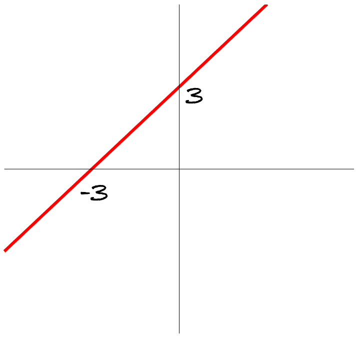
$x = \frac{3}{2}$

5. (i) $y = x + 3$

Gradient $= 1$

When $x = 0$, $y = 3$

When $y = 0$, $x + 3 = 0 \Rightarrow x = -3$

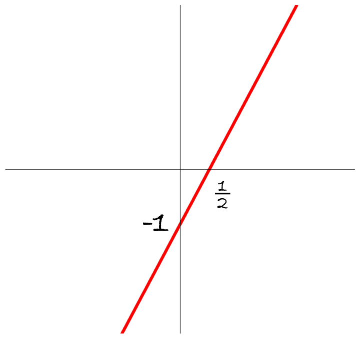


(ii) $y = 2x - 1$

Gradient $= 2$

When $x = 0$, $y = -1$

When $y = 0$, $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$



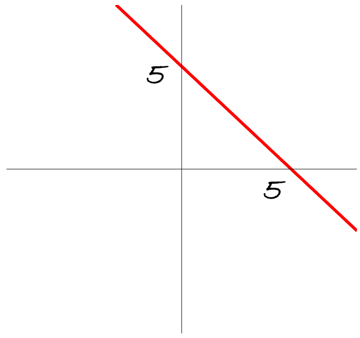
(iii) $x + y = 5$

$y = -x + 5$

Gradient $= -1$

When $x = 0$, $y = 5$

When $y = 0$, $x = 5$



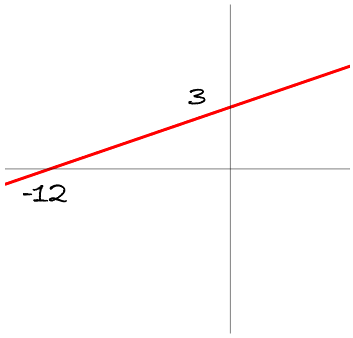
(iv) $4y = x + 12$

$y = \frac{1}{4}x + 3$

Gradient $= \frac{1}{4}$

When $x = 0$, $y = 3$

When $y = 0$, $x + 12 = 0 \Rightarrow x = -12$



(v) $3y + x + 6 = 0$

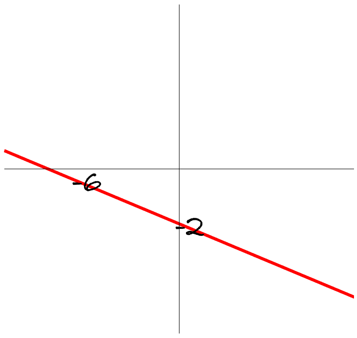
$3y = -x - 6$

$y = -\frac{1}{3}x - 2$

Gradient $= -\frac{1}{3}$

When $x = 0$, $y = -2$

When $y = 0$, $x + 6 = 0 \Rightarrow x = -6$



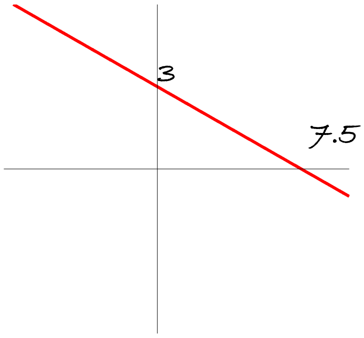
(vi) $5y = 15 - 2x$

$y = 3 - \frac{2}{5}x$

Gradient $= -\frac{2}{5}$

When $x = 0$, $5y = 15 \Rightarrow y = 3$

When $y = 0$, $15 - 2x = 0 \Rightarrow x = 7.5$



6. (a) Gradient $= 1$, y-intercept $= 2$

Equation of line is $y = x + 2$

(b) Gradient $= \frac{1}{2}$, y-intercept $= -1$

Equation of line is $y = \frac{1}{2}x - 1$

or $2y = x - 2$

(c) Gradient $= -\frac{1}{2}$, y-intercept $= -2$

Equation of line is $y = -\frac{1}{2}x - 2$

or $2y + x + 4 = 0$

(d) Gradient $= -\frac{1}{4}$, y- intercept $= 3$

Equation of line is $y = -\frac{1}{4}x + 3$

or $4y + x = 12$

(e) Gradient $= -\frac{8}{3}$, passes through (-1, 4)

Equation of line is $y - 4 = -\frac{8}{3}(x - (-1))$

$3(y - 4) = -8(x + 1)$

$3y - 12 = -8x - 8$

$3y + 8x = 4$

7. (i) Equation of line is $y - 3 = 4(x - 2)$

$y - 3 = 4x - 8$

$y = 4x - 5$

(ii) Equation of line is $y-(-1)=-\frac{1}{3}(x-4)$

$$3(y+1)=-(x-4)$$
$$3y+3=-x+4$$
$$3y+x=1$$

(iii) Equation of line is $y-(-6)=-\frac{1}{5}(x-(-1))$

$$5(y+6)=-(x+1)$$
$$5y+30=-x-1$$
$$5y+x+31=0$$

8. (i) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{6-2}{1-3}=\dfrac{4}{-2}=-2$

Equation of AB is $y-6=-2(x-1)$

$$y-6=-2x+2$$
$$y+2x=8$$

(ii) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{-1-3}{8-(-2)}=\dfrac{-4}{10}=-\dfrac{2}{5}$

Equation of AB is $y-(-1)=-\frac{2}{5}(x-8)$

$$5(y+1)=-2(x-8)$$
$$5y+5=-2x+16$$
$$5y+2x=11$$

(iii) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{2-(-4)}{-5-7}=\dfrac{6}{-12}=-\dfrac{1}{2}$

Equation of AB is $y-2=-\frac{1}{2}(x-(-5))$

$$2(y-2)=-(x+5)$$
$$2y-4=-x-5$$
$$2y+x+1=0$$

(iv) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{-5-1}{-3-5}=\dfrac{-6}{-8}=\dfrac{3}{4}$

Equation of AB is $y-(-5)=\frac{3}{4}(x-(-3))$

$$4(y+5)=3(x+3)$$
$$4y+20=3x+9$$
$$4y=3x-11$$

9. Let the triangle be ABC.

Let A be the intersection point of $y+3x=11$ and $3y=x+3$.

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $3y=x+3$ gives $3(11-3x)=x+3$

$$33-9x=x+3$$
$$30=10x$$
$$x=3$$

When $x=3$, $y=11-3\times3=2$

The coordinates of A are (3, 2).

Let B be the intersection point of $3y=x+3$ and $7y+x=37$

$3y=x+3 \quad \Rightarrow x=3y-3$

Substituting into $7y+x=37$ gives $7y+3y-3=37$

$$10y=40$$
$$y=4$$

When $y=4$, $x=3\times4-3=9$

The coordinates of B are (9, 4).

Let C be the intersection point of $7y+x=37$ and $y+3x=11$

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $7y+x=37$ gives $7(11-3x)+x=37$

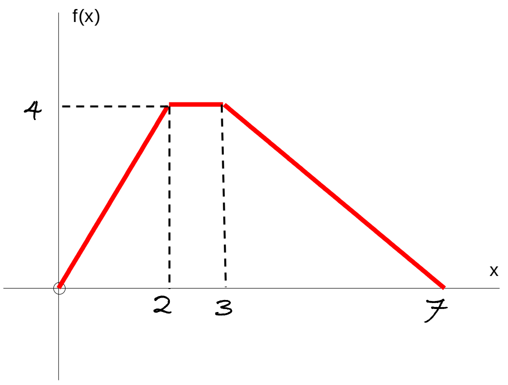
$$77-21x+x=37$$
$$40=20x$$
$$x=2$$

When $x=2$, $y=11-3\times2=5$

The coordinates of C are (2, 5).

The vertices of the triangle are (3, 2), (9, 4) and (2, 5).

10.



11. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{3(x+h)^2-(x+h)-(3x^2-x)}{h}$

$$\begin{aligned}
&=\frac{3(x+h)^2-h-3x^2}{h}\\
&=\frac{3\left\{(x+h)^2-x^2\right\}-h}{h}\\
&=\frac{3(x+h-x)\,(x+h+x)-h}{h}\\
&=\frac{3h(2x+h)-h}{h}\\
&=3(2x+h)-1\\
&=6x+3h-1
\end{aligned}$$

12. One possible function is $f(x)=x^2+2$.