

## Section 1: Straight lines

### Solutions to Exercise

1. (a) (i) Gradient of AB =  $\frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - 4}{3 - 7} = \frac{-3}{-4} = \frac{3}{4}$ .

(ii) Gradient  $m$  of perpendicular line satisfies  $m \times \frac{3}{4} = -1$   
so gradient of perpendicular line =  $-\frac{4}{3}$ .

(iii) Midpoint of AB =  $\left(\frac{3+7}{2}, \frac{1+4}{2}\right) = (5, 2.5)$

(iv) Distance AB =  $\sqrt{(3-7)^2 + (1-4)^2}$   
=  $\sqrt{16+9}$   
=  $\sqrt{25}$   
= 5

(v) Distance from A to B in x-direction =  $7 - 3 = 4$   
x-coordinate of C =  $3 + \frac{3}{5} \times 4 = 3 + 2.4 = 5.4$   
Distance from A to B in y-direction =  $4 - 1 = 3$   
y-coordinate of C =  $1 + \frac{3}{5} \times 3 = 1 + 1.8 = 2.8$   
C = (5.4, 2.8)

(b) (i) Gradient of AB =  $\frac{y_1 - y_2}{x_1 - x_2} = \frac{9 - (-1)}{-2 - 3} = \frac{10}{-5} = -2$ .

(ii) Gradient  $m$  of perpendicular line satisfies  $m \times -2 = -1$   
so gradient of perpendicular line =  $\frac{1}{2}$ .

(iii) Midpoint of AB =  $\left(\frac{-2+3}{2}, \frac{9+(-1)}{2}\right) = (0.5, 4)$

(iv) Distance AB =  $\sqrt{(-2-3)^2 + (9-(-1))^2}$   
=  $\sqrt{25+100}$   
=  $\sqrt{125}$   
=  $5\sqrt{5}$

(v) Distance from A to B in x-direction =  $3 - (-2) = 5$

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$$x\text{-coordinate of } C = -2 + \frac{3}{5} \times 5 = -2 + 3 = 1$$

$$\text{Distance from } A \text{ to } B \text{ in } y\text{-direction} = -1 - 9 = -10$$

$$y\text{-coordinate of } C = 9 + \frac{3}{5} \times -10 = 9 - 6 = 3$$

$$C = (1, 3)$$

$$2. \text{ (i) } M = \left( \frac{2 + (-4)}{2}, \frac{-1 + 8}{2} \right) = (-1, 3.5)$$

$$\text{(ii) Distance from } P \text{ to } Q \text{ in } x\text{-direction} = -4 - 2 = -6$$

$$x\text{-coordinate of } R = 2 + \frac{1}{4} \times -6 = 2 - 1.5 = 0.5$$

$$\text{Distance from } P \text{ to } Q \text{ in } y\text{-direction} = 8 - (-1) = 9$$

$$y\text{-coordinate of } R = -1 + \frac{1}{4} \times 9 = -1 + 2.25 = 1.25$$

$$C = (0.5, 1.25)$$

$$\text{(iii) Distance from } P \text{ to } Q \text{ in } x\text{-direction} = -4 - 2 = -6$$

$$x\text{-coordinate of } S = 2 + \frac{7}{10} \times -6 = 2 - 4.2 = -2.2$$

$$\text{Distance from } P \text{ to } Q \text{ in } y\text{-direction} = 8 - (-1) = 9$$

$$y\text{-coordinate of } S = -1 + \frac{7}{10} \times 9 = -1 + 6.3 = 5.3$$

$$C = (-2.2, 5.3)$$

$$3. \text{ (i) Gradient of } AB = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - y}{3 - 6} = \frac{1 - y}{-3}$$

$$\text{Gradient of } AB = 2 \Rightarrow \frac{1 - y}{-3} = 2$$

$$\Rightarrow 1 - y = -6$$

$$y = 7$$

$$\text{(ii) Distance } AB \text{ is } 5$$

$$\sqrt{(3 - 6)^2 + (1 - y)^2} = 5$$

$$9 + (1 - y)^2 = 25$$

$$(1 - y)^2 = 16$$

$$1 - y = \pm 4$$

$$y = 1 - 4 \text{ or } 1 + 4$$

$$y = -3 \text{ or } 5$$

$$\text{(iii) If } A, B \text{ and } C \text{ are collinear, gradient of } AB = \text{gradient of } AC.$$

$$\text{Gradient of } AC = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - (-2)}{3 - 12} = \frac{3}{-9} = -\frac{1}{3}$$

$$\text{From (i), gradient of } AB = \frac{1 - y}{-3}$$

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$$\frac{1-y}{-3} = -\frac{1}{3}$$

$$1-y=1$$

$$y=0$$

(iv) If AB is perpendicular to BC, then  $\text{grad AB} \times \text{grad BC} = -1$

From (i), gradient of AB =  $\frac{1-y}{-3}$

Gradient of BC =  $\frac{y_1 - y_2}{x_1 - x_2} = \frac{y - (-2)}{6 - 12} = \frac{y+2}{-6}$

$$\frac{1-y}{-3} \times \frac{y+2}{-6} = -1$$

$$(1-y)(y+2) = -18$$

$$2 - y - y^2 = -18$$

$$y^2 + y - 20 = 0$$

$$(y+5)(y-4) = 0$$

$$y = -5 \text{ or } y = 4$$

(v) Length AB = length BC

$$\sqrt{(3-6)^2 + (1-y)^2} = \sqrt{(6-12)^2 + (y-(-2))^2}$$

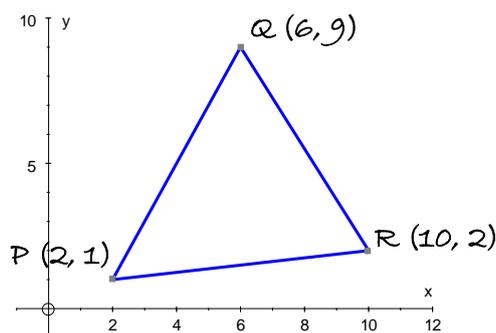
$$9 + (1-y)^2 = 36 + (y+2)^2$$

$$1 - 2y + y^2 = 27 + y^2 + 4y + 4$$

$$0 = 6y + 30$$

$$y = -5$$

4. (i)



(ii)  $PQ = \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

$$PR = \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$$

$$QR = \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ.

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Let M be the midpoint of PQ

$$M = \left( \frac{2+6}{2}, \frac{1+9}{2} \right) = (4, 5)$$

$$\text{Height of triangle is } MR = \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times PQ \times MR \\ &= \frac{1}{2} \sqrt{80} \sqrt{45} \\ &= \frac{1}{2} \sqrt{16 \times 5} \sqrt{9 \times 5} \\ &= \frac{1}{2} \times 4\sqrt{5} \times 3\sqrt{5} \\ &= 6 \times 5 \\ &= 30 \end{aligned}$$

$$5. \text{ Gradient of EF} = \frac{3 - (-1)}{1 - 2} = \frac{4}{-1} = -4$$

$$\text{Gradient of FG} = \frac{5 - 3}{3 - 1} = \frac{2}{2} = 1$$

$$\text{Gradient of GH} = \frac{1 - 5}{4 - 3} = \frac{-4}{1} = -4$$

$$\text{Gradient of EH} = \frac{1 - (-1)}{4 - 2} = \frac{2}{2} = 1$$

EF is parallel to GH and FG is parallel to EH  
so EFGH is a parallelogram.

$$6. \text{ (i) Gradient of } y = 4x - 1 \text{ is } 4$$

Gradient of parallel line = 4

$$\text{Equation of line is } y - 3 = 4(x - 2)$$

$$y - 3 = 4x - 8$$

$$y = 4x - 5$$

$$\text{(ii) Gradient of } y = 2x + 7 \text{ is } 2$$

Gradient of perpendicular line is  $-\frac{1}{2}$

$$\text{Equation of line is } y - 2 = -\frac{1}{2}(x - 1)$$

$$2(y - 2) = -(x - 1)$$

$$2y - 4 = -x + 1$$

$$2y + x = 5$$

$$\text{(iii) } 3y + x = 10 \Rightarrow y = -\frac{1}{3}x + \frac{10}{3}$$

Gradient is  $-\frac{1}{3}$

Gradient of parallel line is  $-\frac{1}{3}$

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$$\text{Equation of line is } y - (-1) = -\frac{1}{3}(x - 4)$$

$$3(y + 1) = -(x - 4)$$

$$3y + 3 = -x + 4$$

$$3y + x = 1$$

$$(iv) \quad 3x + 4y = 12 \Rightarrow y = -\frac{3}{4}x + 3$$

$$\text{Gradient is } -\frac{3}{4}$$

$$\text{Gradient of perpendicular line is } \frac{4}{3}$$

$$\text{Equation of line is } y - 0 = \frac{4}{3}(x - (-3))$$

$$3y = 4(x + 3)$$

$$3y = 4x + 12$$

$$(v) \quad x + 5y + 8 = 0 \Rightarrow y = -\frac{1}{5}x - \frac{8}{5}$$

$$\text{Gradient is } -\frac{1}{5}$$

$$\text{Gradient of parallel line is } -\frac{1}{5}$$

$$\text{Equation of line is } y - (-6) = -\frac{1}{5}(x - (-1))$$

$$5(y + 6) = -(x + 1)$$

$$5y + 30 = -x - 1$$

$$5y + x + 31 = 0$$

$$7. (i) \quad \text{Gradient of } AB = \frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - 2}{1 - 3} = \frac{4}{-2} = -2$$

$$\text{Gradient of perpendicular bisector} = \frac{1}{2}$$

$$\text{Midpoint of } AB = \left( \frac{1 + 3}{2}, \frac{6 + 2}{2} \right) = (2, 4)$$

$$\text{Equation of perpendicular bisector is } y - 4 = \frac{1}{2}(x - 2)$$

$$2y - 8 = x - 2$$

$$2y = x + 6$$

$$(ii) \quad \text{Gradient of } AB = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-1 - 3}{8 - (-2)} = \frac{-4}{10} = -\frac{2}{5}$$

$$\text{Gradient of perpendicular bisector} = \frac{5}{2}$$

$$\text{Midpoint of } AB = \left( \frac{8 + (-2)}{2}, \frac{-1 + 3}{2} \right) = (3, 1)$$

$$\text{Equation of perpendicular bisector is } y - 1 = \frac{5}{2}(x - 3)$$

$$2y - 2 = 5x - 15$$

$$2y = 5x - 13$$

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$$(iii) \text{ Gradient of } AB = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - (-4)}{-5 - 7} = \frac{6}{-12} = -\frac{1}{2}$$

Gradient of perpendicular bisector = 2

$$\text{Midpoint of } AB = \left( \frac{-5 + 7}{2}, \frac{2 + (-4)}{2} \right) = (1, -1)$$

$$\begin{aligned} \text{Gradient of perpendicular bisector is } y - (-1) &= 2(x - 1) \\ y + 2 &= 2x - 2 \\ y &= 2x - 4 \end{aligned}$$

$$(iv) \text{ Gradient of } AB = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5 - 1}{-3 - 5} = \frac{-6}{-8} = \frac{3}{4}$$

Gradient of perpendicular bisector =  $-\frac{4}{3}$

$$\text{Midpoint of } AB = \left( \frac{-3 + 5}{2}, \frac{-5 + 1}{2} \right) = (1, -2)$$

$$\begin{aligned} \text{Gradient of perpendicular bisector is } y - (-2) &= -\frac{4}{3}(x - 1) \\ 3y + 6 &= -4x + 4 \\ 3y + 4x + 2 &= 0 \end{aligned}$$

$$8. (i) \text{ Midpoint of } EF = \left( \frac{2 + 4}{2}, \frac{5 + 1}{2} \right) = (3, 3)$$

$$\text{Midpoint of } FG = \left( \frac{4 + (-2)}{2}, \frac{1 + (-3)}{2} \right) = (1, -1)$$

$$\text{Midpoint of } EG = \left( \frac{2 + (-2)}{2}, \frac{5 + (-3)}{2} \right) = (0, 1)$$

Median from midpoint of EF (3, 3) to G (-2, -3)

$$\text{Gradient of median} = \frac{-3 - 3}{-2 - 3} = \frac{-6}{-5} = \frac{6}{5}$$

$$\text{Equation of median is } y - 3 = \frac{6}{5}(x - 3)$$

$$5(y - 3) = 6(x - 3)$$

$$5y - 15 = 6x - 18$$

$$5y = 6x - 3$$

Median from midpoint of FG (1, -1) to E (2, 5)

$$\text{Gradient of median} = \frac{5 - (-1)}{2 - 1} = \frac{6}{1} = 6$$

$$\text{Equation of median is } y - (-1) = 6(x - 1)$$

$$y + 1 = 6x - 6$$

$$y = 6x - 7$$

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Median from midpoint of  $EG$   $(0, 1)$  to  $F$   $(4, 1)$   $B(9, 4)$

$$\text{Gradient of median} = \frac{1-1}{4-0} = \frac{0}{4} = 0$$

$$\text{Equation of median is } y = 1$$

(ii) Equation of first median is  $5y = 6x - 3$

$$\text{Substituting } x = \frac{4}{3} \text{ gives } 5y = 6 \times \frac{4}{3} - 3 = 8 - 3 = 5$$

$$y = 1$$

so  $(\frac{4}{3}, 1)$  lies on the median.

Equation of second median is  $y = 6x - 7$

$$\text{Substituting } x = \frac{4}{3} \text{ gives } y = 6 \times \frac{4}{3} - 7 = 8 - 7 = 1$$

so  $(\frac{4}{3}, 1)$  lies on the median.

Equation of third median is  $y = 1$ , so  $(\frac{4}{3}, 1)$  lies on the median.

9. (i)  $B$  is the intersection point of  $y = 4x - 3$  and  $y = 2x + 1$

$$4x - 3 = 2x + 1$$

$$2x = 4$$

$$x = 2$$

$$\text{When } x = 2, y = 4 \times 2 - 3 = 5$$

The coordinates of  $B$  are  $(2, 5)$ .

(ii)  $AD$  is parallel to  $BC$ , so  $AD$  has gradient 2.

$AD$  passes through the point  $(3, 9)$ .

$$\text{Equation of } AD \text{ is } y - 9 = 2(x - 3)$$

$$y - 9 = 2x - 6$$

$$y = 2x + 3$$

(iii)  $CD$  is parallel to  $AB$ , so  $CD$  has gradient 4.

$CD$  passes through the point  $(7, 15)$ .

$$\text{Equation of } CD \text{ is } y - 15 = 4(x - 7)$$

$$y - 15 = 4x - 28$$

$$y = 4x - 13$$

(iv)  $D$  is the intersection point of  $AD$  and  $CD$ .

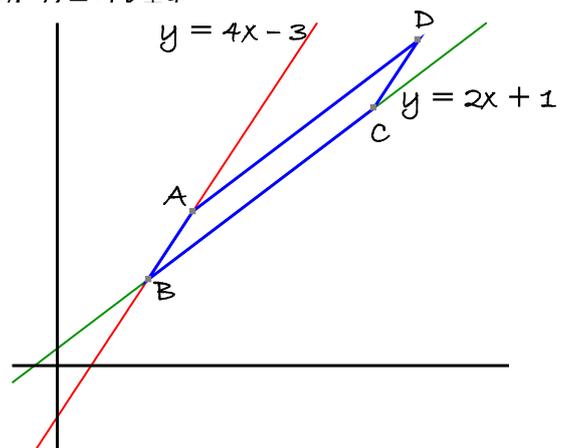
$$2x + 3 = 4x - 13$$

$$16 = 2x$$

$$x = 8$$

$$\text{When } x = 8, y = 2 \times 8 + 3 = 19$$

The coordinates of  $D$  are  $(8, 19)$ .



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10. The diagonals of the rhombus cross at the midpoint of  $OB$ , ie at  $(3, 2)$ .

$OB$  has gradient  $\frac{2}{3}$ .

$AD$  is perpendicular to  $OB$  so has gradient  $-1.5$ .

The equation of  $AD$  is  $(y - 2) = -1.5(x - 3)$

$$y = -1.5x + 6.5$$

$D$  is on the  $x$ -axis so at  $D: 0 = -1.5x + 6.5$

$$x = \frac{6.5}{1.5} = 4\frac{1}{3}$$

$AB = OD$  and  $AB$  is parallel to the  $x$ -axis so  $A$  is the point  $\left(1\frac{2}{3}, 4\right)$ .

11. The gradient of  $AB$  is  $-\frac{5}{2}$ .

$AD$  is perpendicular to  $AB$  and so has gradient  $\frac{2}{5}$ .

Point  $D$  has coordinates  $(7, 2)$ .

The coordinates of  $E$  are  $\left(\frac{2 \times 2 + 7}{3}, \frac{2 \times 0 + 2}{3}\right) = \left(\frac{11}{3}, \frac{2}{3}\right)$

The gradient of  $BE$  is  $-\frac{13/3}{11/3} = -\frac{13}{11}$

$BE$  is the line  $y = -\frac{13}{11}x + 5$ . This crosses the  $x$ -axis at  $\left(\frac{55}{13}, 0\right)$ .