

## Section 3: Exponentials functions

### Notes and Examples

These notes contain subsections on

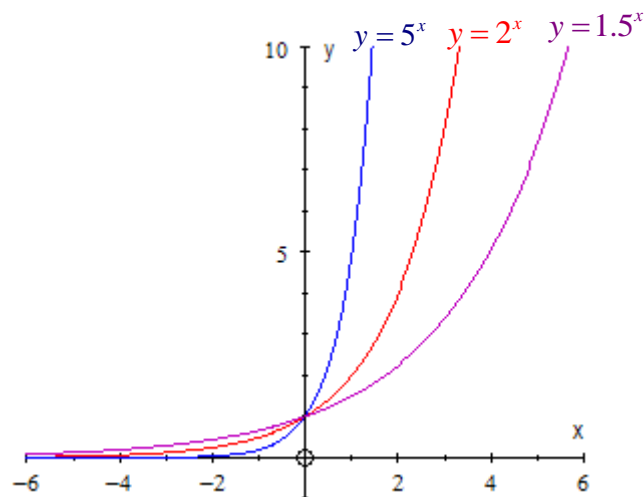
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### Exponential functions

An exponential function is any function of the form  $y = a^x$ . The table below gives the value of  $y$  for particular values of  $x$ , for  $a = 1.5, 2$  and  $5$ .

$x$	-4	-2	-1	0	1	2	3	4	5
$5^x$	0.0016	0.04	0.2	1	5	25	125	625	3125
$2^x$	0.0625	0.25	0.5	1	2	4	8	16	32
$1.5^x$	0.197530864	0.444444444	0.666666667	1	1.5	2.25	3.375	5.0625	7.59375

The graphs below show these exponential functions. Compare these to the values in the table above to make sure you understand the relationship between them.



Here is a simple example with an exponential function



#### Example 1

Let  $y = 3^x$

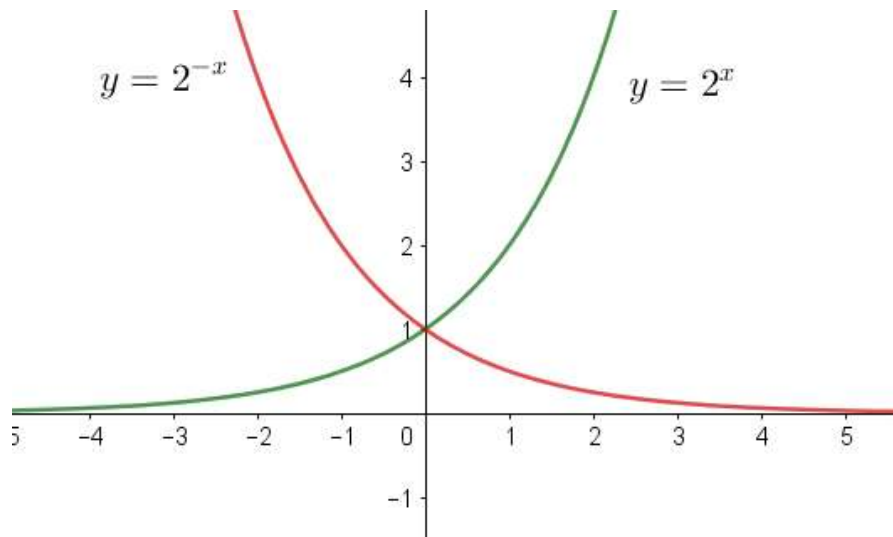
- (i) What is  $y$  when  $x = -2, 1, 3$ ?
- (ii) For which values of  $x$  is  $y = \frac{1}{3}, 1, 9$ ?

**Solution**

- (i)  $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ ,  $3^1 = 3$ ,  $3^3 = 27$  so the values of  $y$  are  $\frac{1}{9}$ , 3 and 27 respectively.
- (ii)  $3^{-1} = \frac{1}{3}$ ,  $3^0 = 1$ ,  $3^2 = 9$  so the values of  $x$  are -1, 0 and 2 respectively.



In the diagram below you see the graphs  $y = 2^x$  and  $y = 2^{-x}$ . Note that each is a reflection of the other.



$y = 2^x$  increases as  $x$  increases whereas  $y = 2^{-x}$  decreases. This is true of  $y = a^x$  and  $y = a^{-x}$  where  $a$  is positive real number. You may wish to investigate this using graphing software with sliders.

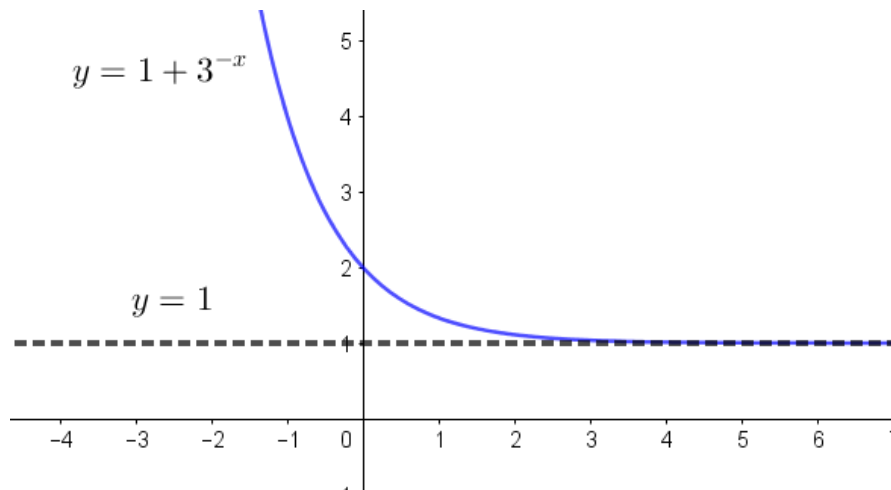
## Applications of Exponential functions

Many real-life situations can be modelled by exponential functions. The growth of a population (e.g. of people, animals or bacteria) can be modelled by an exponential function. A model like this might take the form  $y = c \times a^{kt}$ . This type of model is called **exponential growth**.

In an exponential growth model, the quantity being modelled continues to increase, at an ever-increasing rate. In a real-life situation such as the growth of a population, the model will eventually break down, since other factors such as overcrowding or limited resources will affect the growth of the population.

Another type of model is **exponential decay**, in which something decreases exponentially. A model like this might take the form  $y = c \times a^{-kt}$ . Exponential decay could model the temperature of a cooling liquid, or the mass of a radioactive isotope remaining.

In an exponential decay model, the quantity being modelled decreases at a rate which becomes slower and slower. The quantity will approach a limiting value, but never quite reach it. For example, the graph below shows the curve  $y = 1 + 3^{-x}$ . The graph approaches the line  $y = 1$  as  $x$  becomes large.



### Example 2

A cup of coffee that has just been made at time  $t = 0$ , cools as modelled by the formula below, where  $T$  in  $^{\circ}\text{C}$  is the temperature of the coffee at time  $t$  minutes.

$$T = 18 + 54 \times 2.7^{-t/27}$$

What is the temperature of the coffee

- (i) when it has just been made?
- (ii) one minute after it has been made?
- (iii) one hour after it has been made?

### Solution

(i) At time  $t = 0$  mins, the temperature of the coffee is  $T = 18 + 54 \times 2.7^0 = 72^{\circ}\text{C}$

(ii) At time  $t = 1$  mins, the temperature of the coffee is

$$T = 18 + 54 \times 2.7^{-1/27} = 70.049591^{\circ}\text{C}.$$

(iii) At time  $t = 60$  mins, the temperature of the coffee is

$$T = 18 + 54 \times 2.7^{-60/27} = 23.940290^{\circ}\text{C}.$$

